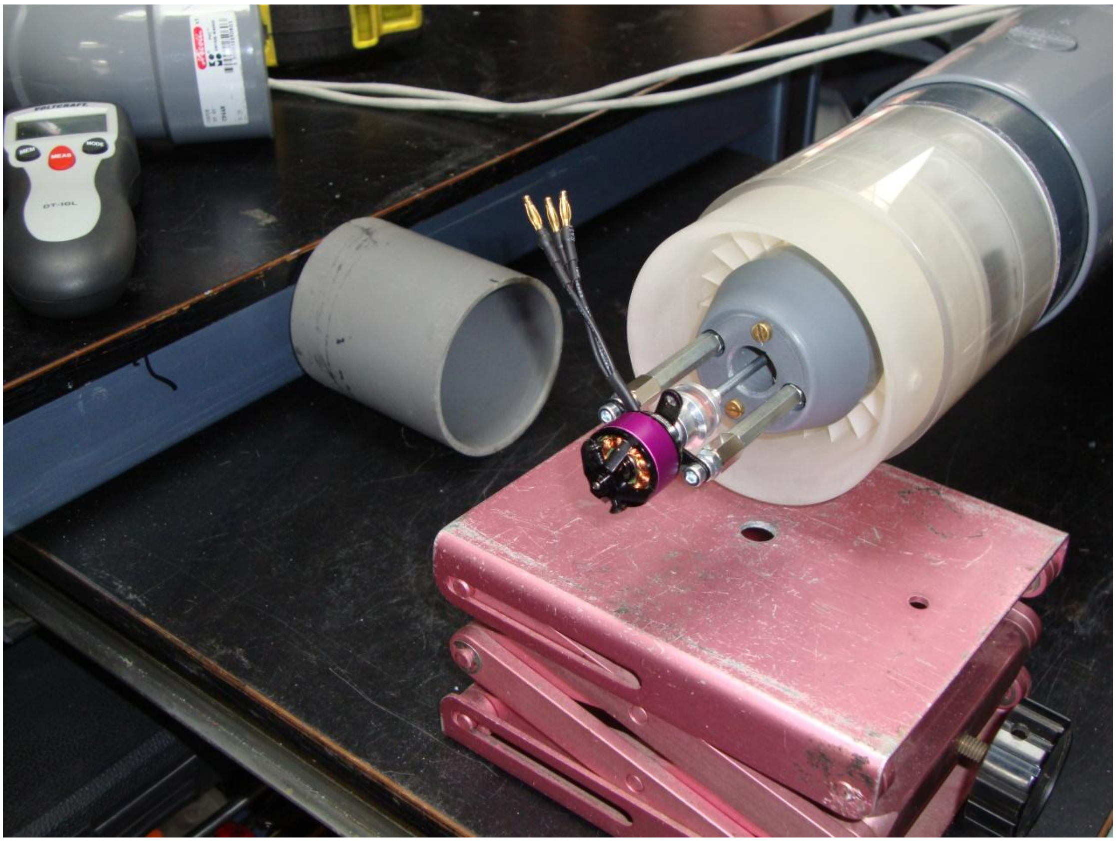
Turbine Design for Thermo-acoustic Generator

Design of a bi-directional turbine to convert acoustic power into electricity



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Version 5.0

The design of a bi-directional turbine to convert acoustic power into electricity. First, study is done on thermo-acoustic systems which produce the acoustic power, followed by assessing 3 different types of bi-directional turbines. After trade-off study the best bi-directional turbine type is chosen and designed in detail with an aim to produce 50W of electricity. The final design is built using 3D-print techniques and finally tested.

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Preface

This report is a result of my bachelor thesis which I worked on for the past 5 months to finish my study Aeronautical Engineering at INHolland University of Applied Sciences of Delft. During this thesis I worked on the development of a bi-directional turbine to convert acoustic power into electricity.

Before I started this project I was already interested in sustainable and innovative projects. Via a training course on sustainable entrepreneurship I got in contact with FACT-foundation, that is exploring the opportunities of using thermo-acoustics for energy production in developing countries. Although thermo-acoustics is a new and unknown way to produce sustainable energy I was interested in it and therefore I chose to work on this project for my bachelor thesis.

I would like to thank my company supervisor Winfried Rijssenbeek who gave me the opportunity to work on an innovative project like this. During the project I was able to apply my knowledge gained during my study and to study more about thermo-acoustics. I also would like to thank Kees de Blok from Aster Thermoakoestische Systemen. As a specialist in thermo-acoustics, he learned me more about the ins and outs of thermo-acoustics, he built the prototype of the thermo-acoustic system in which the turbine is implemented. During the project I had some delays with the production of the prototype, but I was always supported by Winfried Rijssenbeek, Kees de Blok and Daphne Bantjes.

Summary

In developing countries there is a lack of electricity. FACT-foundation is developing bio-energy solutions for these countries. Often in developing countries wood is used in cooking stoves. A part of the heat produced by cooking stoves can be used to generate electricity. This can be done using thermo-acoustics. A thermo-acoustic system uses the heat produced by cooking stoves to generate sound waves. These sound waves are a form of energy which can be converted into electricity by using a generator. In small communities in developing countries there is a lack of electricity, so for these communities a small thermo-acoustic system can be developed to produce electricity on a household scale.

Previously, a linear alternator is used to convert the acoustic power into electricity. Due to its linear motion a lot of energy is lost, which means a low efficiency. This efficiency can be improved by increasing the size of the alternator, but this results in an alternator which is too big and too heavy. Another option is a bi-directional turbine which uses a rotational motion to convert acoustic power into electricity. A simple test performed by Aster showed that it is more reasonable to use a turbine instead of a linear alternator to convert the energy into electricity. The main question is: What is the most efficient turbine type to convert sound waves into electricity? Next, the goal is to design a turbine which has an output power of 50 Watt of electricity. Besides that, the bi-directional turbine should be simple and as cheap as possible.

Three different bi-directional turbine types are considered: a Wells turbine, a radial and an axial impulse turbine. After trade-off study it is found out that the axial impulse turbine is the best suitable turbine for the thermo-acoustic system. Due to its high efficiency at a wide range of velocities, good self-starting characteristics and its easy construction into the thermo-acoustic system. A final design is drawn in CATIA based upon theoretical calculations. Next, a prototype is made using 3D-print techniques and tested to compare the test results with the theoretical calculations.

Concluding, the most efficient turbine type to convert acoustic power into electricity in the thermo-acoustic system design for FACT is an axial impulse turbine. The obtained test-results meet with the theoretical assumptions, meanwhile an output of 50 Watt of electricity is not possible (up to now only 32.5 Watt of electricity) with a thermo-acoustic system of this size. Still, the turbine efficiency and the output power can be improved by optimizing the design of the axial impulse turbine. CFD-analysis can be performed to find the places on the turbine where losses can be reduced. Moreover, it is recommended to test the axial impulse turbine in the thermo-acoustic system, because during this project this system was not finished yet.

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List of terms

Acoustic impedance meter	An acoustic impedance meter generates a sound wave and measures the acoustic power that goes into the turbine.
Adiabatic index	The ratio between the specific heat at constant pressure (c_p) and the specific heat at constant volume (c_v), c_p /c_v. For the adiabatic index of air a value of 1.4 can be assumed.
Bi-directional turbine	A bi-directional turbine turns always in the same direction regardless the direction of the moving air.
Bouncing space	Bouncing space is an extra chamber that is attached to the thermo-acoustic system. In this chamber the pressure is the same as in the rest of the thermo-acoustic system.
Brushless outrunner	Type of generator which is used to generate electricity. The inner part is fixed and consists out of a coil, while the outer part has a magnet and can rotate. An axis is attached to the outer part of the outrunner which is attached to the turbine.
Compression	During compression the volume in the medium is decreasing.
CATIA	Program that is used to make 3D-drawings.
Engine input temperature	The temperature of a fluid when it just left the regenerator (highest temperature in the system).
Guide vanes	Guide vanes are fixed airfoils that direct the air into the moving turbine blades in order to increase the turbine efficiency
Heat exchanger	At high temperatures a heat exchanger adds heat to the thermo-acoustic system (Hot Heat Exchanger) and at low temperatures a heat exchanger extracts heat form the thermo-acoustic system (Ambient Heat Exchanger).
Heat rejection temperature	The temperature of a fluid just before it goes into the regenerator (lowest temperature in the system).
Hub-tip ratio	The ratio between the radius of the hub and the tip of the turbine blade
Oscillating Water Column	An oscillating water column uses a large volume of moving water as a piston in a cylinder. Air is forced out of the column as a wave rises and fresh air is drawn in as the wave falls. This movement of air turns a turbine at the top of the column.

Regenerator	A regenerator consists out of a steel mesh which is placed between both heat exchangers. It is used to create sound waves, by reducing the volume and rising pressure and temperature.
Stereolithography	Type of 3D-printtechnique where the product is built up out of thin layers. In every layer a laser is directed to a liquid. Because of the laser the liquid is cured and so the product can be realized.
Thermo-acoustic generator	A thermo-acoustic generator is a thermo-acoustic system including a generator and a bi-directional turbine.
Thermo-acoustic system	A thermo-acoustic system uses heat to generate acoustic power

List of symbols

Symbol	*Description*	*Unit*
A	Cross-sectional area	m^2
c	Speed of sound	m/s
$\mathbf{C_D}$	Drag coefficient	-
$\mathbf{C_L}$	Lift coefficient	-
D	Drag force	N
$\mathbf{E_b}$	Kinetic energy through blade passage	W
$\mathbf{F_x}$	Axial force	N
$\mathbf{F_\theta}$	Tangential force	N
f	Wave frequency	Hz (s^{-1})
$\mathbf{k_b}$	Blade friction factor	-
L	Lift force	N
m	Mass	kg
$\dot{\mathbf{m}}$	Mass flow rate	kg/s
n	Rotational speed (of the turbine)	Rad/s or rpm
$\mathbf{p_a}$	Pressure amplitude	Pa
$\mathbf{P_t}$	Tangential thrust	N
R	Specific gas constant	J/(kg·K)
r	Turbine radius	m
S	Surface area of a wing (in case of the Wells Turbine the surface of the blade)	m^2
T	Temperature	K
v	Absolute velocity	m/s
$\mathbf{v_a}$	Velocity amplitude	m/s
$\mathbf{v_{ax}}$	Axial velocity	m/s
$\mathbf{v_b}$	Blade velocity	m/s
$\mathbf{v_r}$	Relative velocity	m/s
$\mathbf{v_\omega}$	Tangential velocity	m/s
U	Voltage	V
V	Volume	m^3
$\mathbf{W_D}$	Blading work/Diagram work	W
$\mathbf{W_G}$	Generator work	W
$\mathbf{Z_0}$	Acoustic impedance	$N\cdot s\cdot m^{-3}$
α	Guide vane angle	°
β	Blade inlet angle	°
$\mathbf{\eta_D}$	Diagram efficiency	-
ρ	Density	kg/m^3
ϕ	Turbine circumference	m

List of abbreviations

Notation	*Description*
°	Degrees (angle)
°C	Degrees Celsius
3D	3-Dimensional
A	Ampere
AHEX	Ambient Heat Exchanger
HHEX	Hot Heat Exchanger
Hz	Hertz
J	Joule
K	Kelvin
kg	Kilograms
kg/s	Kilograms per second (mass flow rate)
m	Meters
m^2	Square meters
m/s	Meters per second
N	Newton
OWC	Oscillating Water Column
Pa	Pascal
rad/s	Radians per second
rpm	Rounds per minute
s	Seconds
V	Voltage
W	Watt

1. Introduction

1.1. Background

FACT-foundation is developing bio-energy solutions for developing countries. In these countries there is a lack of electricity. Often in developing countries wood is used in cooking stoves. A part of the heat produced by cooking stoves can be used to generate electricity. This can be done by using thermo-acoustics. Thermo-acoustics is a general name for a thermodynamic cycle where heat is converted into acoustic power. The advantage of a thermo-acoustic system is that mechanical moving parts are not needed. These moving parts are replaced by compression and displacement of a gas into a powerful acoustic wave. The dynamics of this process are complex, but in practice the implementation is straightforward. Therefore, the production and maintenance costs are low. In collaboration with Aster Thermakoestiche Systemen FACT-Foundation develops a thermo-acoustic generator for people in developing countries who can produce electricity on a household scale using residual heat of a cooking stove.

1.2. Goal of the project

The acoustic power in a thermo-acoustic system is a movement of air that goes back and forth. Previously, a linear (moving) alternator is used to convert acoustic power into electricity. However, a linear alternator is considered too big, too heavy and too expensive. Detailed information about the disadvantages of a linear alternator can be found in chapter 3. According to Aster Thermakoestiche Systemen there is another option to convert acoustic power into electricity, namely a bi-directional turbine which uses a rotating instead of a linear motion. A bi-directional turbine turns always in the same direction regardless the direction of the moving air. In this concept a standard electric generator, which generates the electricity, is driven by the bi-directional turbine.

A Wells Turbine is a type of bi-directional turbine which is developed by Prof. Alan Wells in the 1970's. It is developed for use in Oscillating Water Column (OWC) wave power plants. In this column waves are used to move air. Next to the column a cylinder is placed with a Wells Turbine, behind the turbine there is an open space too. During the process the wind blows from the column through the cylinder to the open space and backwards, as a result of the waves. The turbine starts rotating and drives the generator and this generator generates electricity. A schematic overview of an OWC including a Wells Turbine is shown in Figure 1.1.

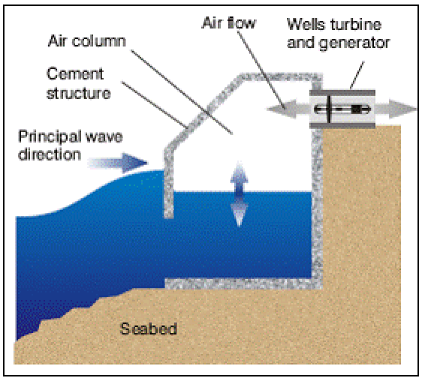


Figure 1.1: Schematic overview of an Oscillating Water Column including a Wells Turbine

The moving air in a OWC due to waves is comparable to the sound waves in a thermo-acoustic system, in terms of converting a linear into a rotating motion using a bi-directional turbine. In the light of the smarter alternatives to the linear alternator the assignment is given to design a bi-directional turbine which converts acoustic power into electricity. The bi-directional turbine drives a generator, which generates the electricity. There are different types of bi-directional turbines considered in this project, so the main question of the project is: 'What is the most efficient bi-directional turbine type to convert acoustic power into electricity?' The three considered bi-directional turbine types are:

1) Wells Turbine
2) Axial Impulse Turbine
3) Radial Impulse Turbine

All these turbines are studied and a selection is made. Additionally, sub-questions can be raised as follows:

- Is it possible to reach an output power of 50 Watt of electricity? Theoretically and in practice.
- What are the required input values for the turbine to convert acoustic power into electricity?
- What is the turbine efficiency of the final design? Theoretically and in practice.
- What is the effect of the wave frequency to the turbine efficiency?

After literature study on the three different bi-directional turbine types a trade-off is made and the best turbine type is chosen, based on selection criteria which are discussed in chapter 5. Next, a detailed design is made of the bi-directional turbine. Then, the final design is made using 3D-printing. Finally, a prototype of the turbine is tested and test results are compared with the theoretical calculations.

1.3. Outline

In chapter 3 the linear alternator is discussed, followed by chapter 2 which is about the thermo-acoustic system. Next, in chapter 4 analysis are performed on bi-directional turbines, while trade-off study is done in the 5th chapter. In chapter 6 a detailed design is made of this turbine and drawn in CATIA. In the 0th chapter test results are analyzed and compared with the theoretical expectations. Finally, a conclusion of this project is given in chapter 0 before the last chapter where recommendations are given.

2. Thermo-acoustic system

In a thermo-acoustic system acoustic power is generated, but how does a bi-directional turbine convert this power into electricity? What are the input values for the turbine? Further, in this chapter the design of the thermo-acoustic system is shown, including its dimensions. The final design of the bi-directional turbine has to fit into the existing thermo-acoustic system.

2.1. Design of the thermo-acoustic system

Aster Thermoakoestische Systemen has designed a thermo-acoustic system for FACT-Foundation to generate 50 Watt electric power. In Figure 2.1 a schematic overview is given of this design. To gain more acoustic power in the thermo-acoustic system the pressure in the system is raised to $4\ bar$. Next, the diameter of the tube $D_t = 84.6\ mm$.

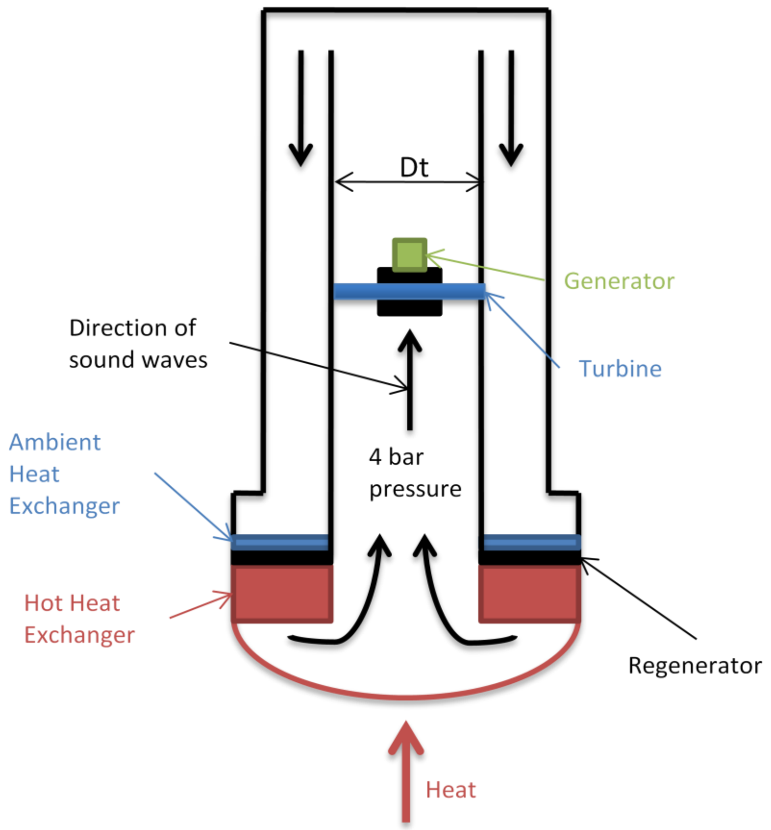


Figure 2.1: Schematic overview of design of the thermo-acoustic system for 50 Watt electricity (left), prototype of the current thermo-acoustic system (right)

In the following part the working principle of the thermo-acoustic system is explained, note that this is a quote from the website of Aster Thermakoestiche Systemen [9]:

Pressure and temperature

Figure 2.2 shows what happens to a single parcel during a thermodynamic cycle. When stimulated by an impulse or vibration the gas will be compressed. As a result the temperature of the parcel will increase. During the next phase when the gas expands the temperature will decrease immediately. A compressed gas parcel tends to release its heat to the environment while an expanding one will extract heat from its surroundings. These characteristics are essential to TA heat pumps because in this way heat can be withdrawn from one location in the system and deposited at another location.

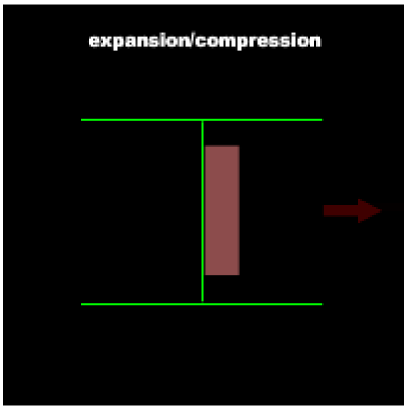


Figure 2.2: A single parcel during a thermodynamic cycle

- *Advanced reading* - Starting at the moment of minimum pressure (t=0) the gas will be compressed by the acoustic wave. In a travelling wave the displacement of the gas is one quarter of a period behind the pressure amplitude. From this the gas at the start of the compression (t=0) is in the equilibrium (middle) position (Uo), shown in Figure 2.3. During compression the gas moves to the left (-U). Because there is a maximal heat transfer (isothermal propagation) heat (Q1) is released to the regenerator left of the equilibrium position. In the second half of the cycle the opposite occurs. During expansion the gas moves to the right (+dU) of the equilibrium position locally extracting heat (Q2) from the regenerator. A complete cycle controlled by a travelling wave therefore includes compression and heat sink (Q1) at the left side (-dU) at a high temperature (T1) followed by expansion and heat extraction (Q2) at the right (+dU) at low temperature (T2).

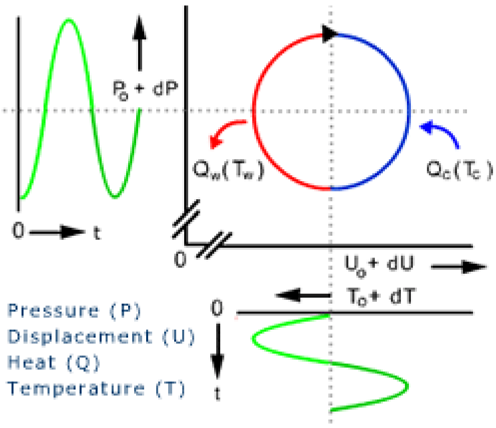


Figure 2.3: Process in the thermoacoustic system

Traveling wave

Imagine a column of gas as a sequence of gas parcels. When at the left side of the column a single impulse or vibration is initiated the following occurs:

- The parcel moves from the left to the right and back. The speed of this movement is called the gas velocity. During this movement the gas will be compressed and expand once. The magnitude of this variation in pressure is the pressure amplitude;
- The temperature of the gas parcel will go up and down;
- The parcel transfers the impulse to the next parcel. The speed at which the impulse is transferred is called the speed of sound or propagation speed.

The last property causes a travelling (longitudinal) wave in the direction of propagation. The number of impulses per second is the operating frequency and is expressed in Hertz (Hz).

2.2. Input values

In Figure 2.4 a graph is shown with the continuously changing pressure in the thermo-acoustic system as a result of the sound wave. The peak values in the graph are the pressure amplitude p_a. These pressure amplitudes cause the acoustic power, nevertheless the acoustic power is not used as an input value for the bi-directional turbine. Still, the pressure amplitude is needed to calculate the input value for the turbine, but this is discussed later in this paragraph.

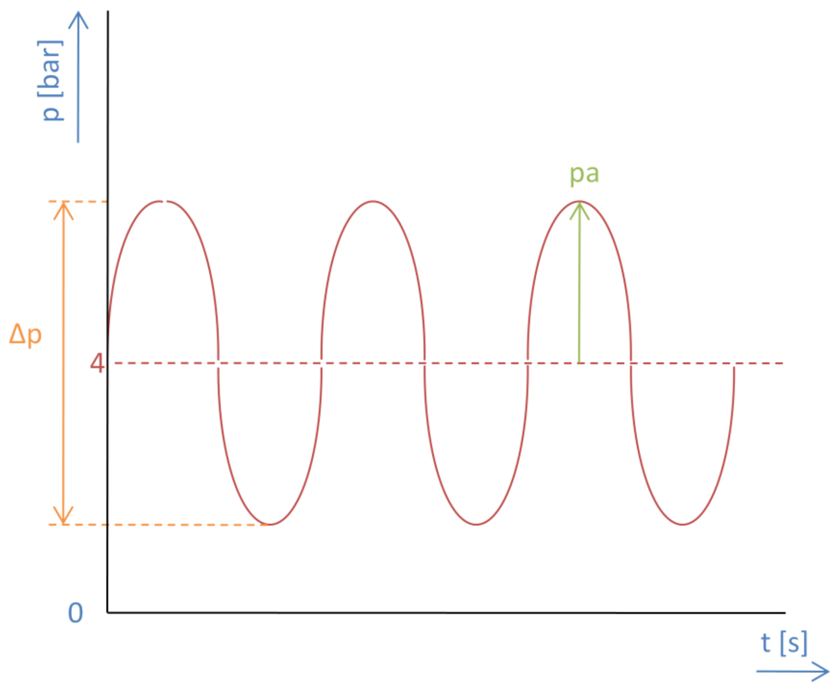


Figure 2.4: Pressure amplitude

In fact, the air in the thermo-acoustic system is vibrating. In other words, the air is moving to the front and back constantly. In Figure 2.5 a graph is shown with the velocity per time unit. The average velocity is $0\,m/s$, but the peak values (velocity amplitudes) can be used to drive a bi-directional turbine.

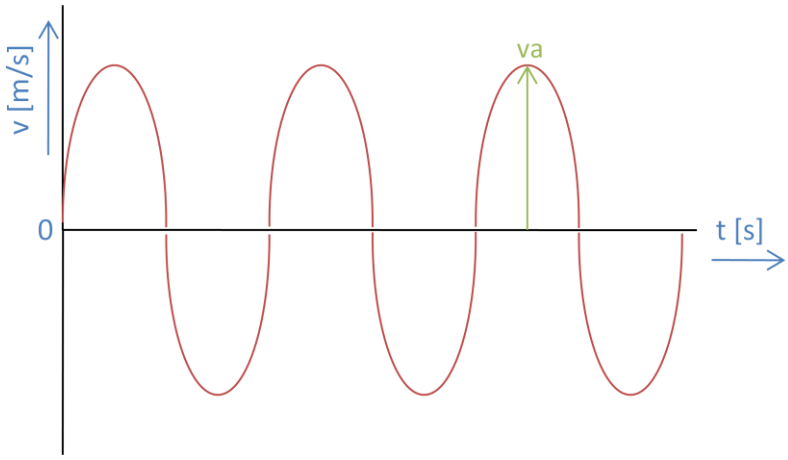


Figure 2.5: Velocity amplitude

To calculate the velocity amplitude, the following formula is used [2]:

$$v_a = \frac{p_a}{Z_0}$$

Where Z_0 is the acoustic impedance, which gives the ratio between pressure and velocity amplitudes. To calculate the acoustic impedance the formula $Z_0 = \rho \cdot c$ is used, where ρ is the density of air in the thermo-acoustic system and c is the speed of sound. Finally, p_a is the pressure amplitude. Aster has designed a thermo-acoustic system with a pressure amplitude which is 5% of the average pressure in the system. Now, the velocity amplitude can be determined with the input values shown in Table 2.1.

Table 2.1: Input values to calculate the velocity amplitude

Input	Symbol	Value	Unit
Pressure amplitude	p_a	18239	Pa
Density	ρ	2.954	kg/m^3
Speed of sound	c	438.2	m/s

This results in a velocity amplitude of $14.09\ m/s$. Still, this is not the input value for the bi-directional turbine. The velocity amplitude is a peak value, but an average velocity is needed, because the peak value (velocity) is not a constant. Therefore, the effective value is calculated [15]:

$$v_0 = \frac{v_a}{\sqrt{2}}$$

Where v_a is the velocity amplitude and v_0 is the average velocity or the effective value of the velocity. The blue line shown in Figure 2.6 is the average velocity. The average velocity v_0 is the input value of a bi-directional turbine. In the designed Thermo-acoustic system $v_0 = 9.96\, m/s$.

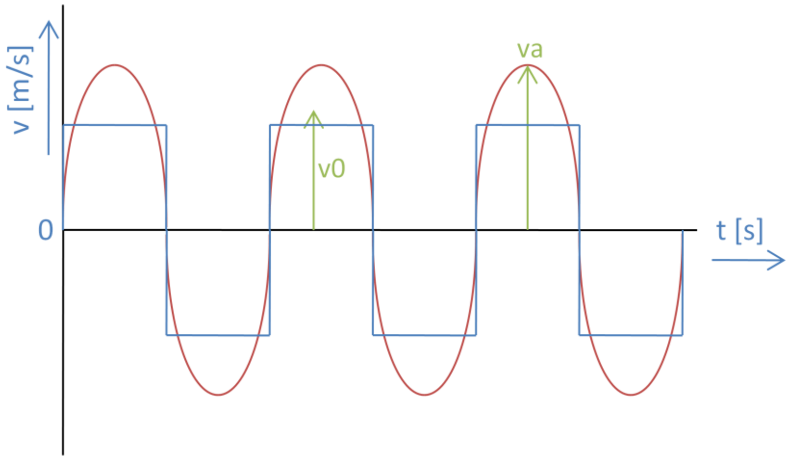


Figure 2.6: Average velocity v_0

2.3. Wave frequency

Aster Thermoakoestische Systemen has designed the thermo-acoustic system with a wave frequency within a range of $10 - 200\ Hz$. The frequency is the time a particle takes to complete one cycle (the period T). A wave frequency is shown in Figure 2.7.

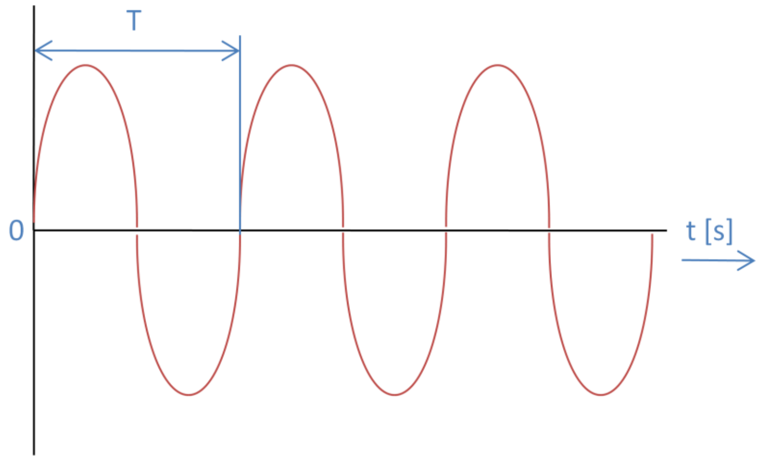


Figure 2.7: Wave frequency

During this project the effect of the wave frequency on the turbine efficiency is studied. Therefore, the following formula is used:

$$f = \frac{1}{T}$$

Where f is the wave frequency in Hertz and T is the period in seconds. Half the period is the time a particle takes to go through the turbine in one direction. In the final design, discussed in chapter 6, the effect of the wave frequency on the turbine efficiency is further elaborated.

2.4. Conclusion

A bi-directional turbine uses the average velocity v_0 of the velocity amplitude v_1 as an input value to convert the acoustic power into electricity. The average velocity is $9.96\,m/s$ while the average pressure in the designed thermo-acoustic system is $4\ bar$. Besides, the tube diameter in the thermo-acoustic system is $84.6\ mm$. This means that the turbine diameter is limited to that diameter if no further complications are made.

3. Linear alternator

Before this project started a linear alternator is used to convert acoustic power into electricity. In this chapter its functionary is explained. A linear alternator uses a linear motion to convert the acoustic power into electricity. In Figure 3.1 a schematic overview is given of a linear alternator, where Z_p is the acoustic power. Moreover, a magnet and a spring are attached to the piston. Finally, a coil is placed around the piston. During operation the piston moves from the left to the right, due to the moving air caused by the sound waves. To generate electricity a voltage is needed, which is generated by the motion of the magnet inside the coil. The voltage is calculated with the following formula [2]:

$$U = v \cdot B \cdot l$$

Where v is the velocity of the coil or the magnet, B is the magnetic field strength of the magnet and l length of the winding of the coil. To increase the voltage the velocity v needs to be increased, although this is limited by the counteracting force of the spring. This spring reaction is one of the disadvantages of a linear alternator. Another option to increase the voltage is to increase the product $B \cdot l$. However, this requires stronger, heavier and more expensive magnets. At the moment when the magnet changes the moving direction it comes to a rest. So, no energy is generated at this point. A solution to this problem is to use a generator with a rotating mechanism instead of linear motion. This generator can be driven by a bi-directional turbine. Because of the constant rotation of this turbine in one direction less energy is lost than in a linear alternator. In Figure 3.2 a thermo-acoustic generator is shown with 4 linear alternators.

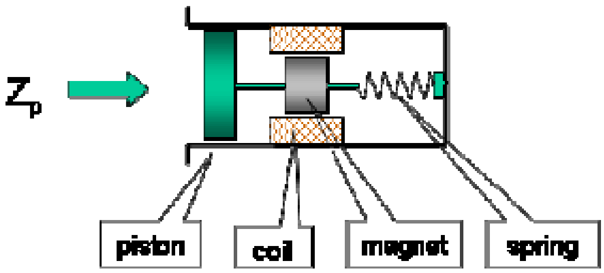


Figure 3.1: Schematic overview of a linear alternator

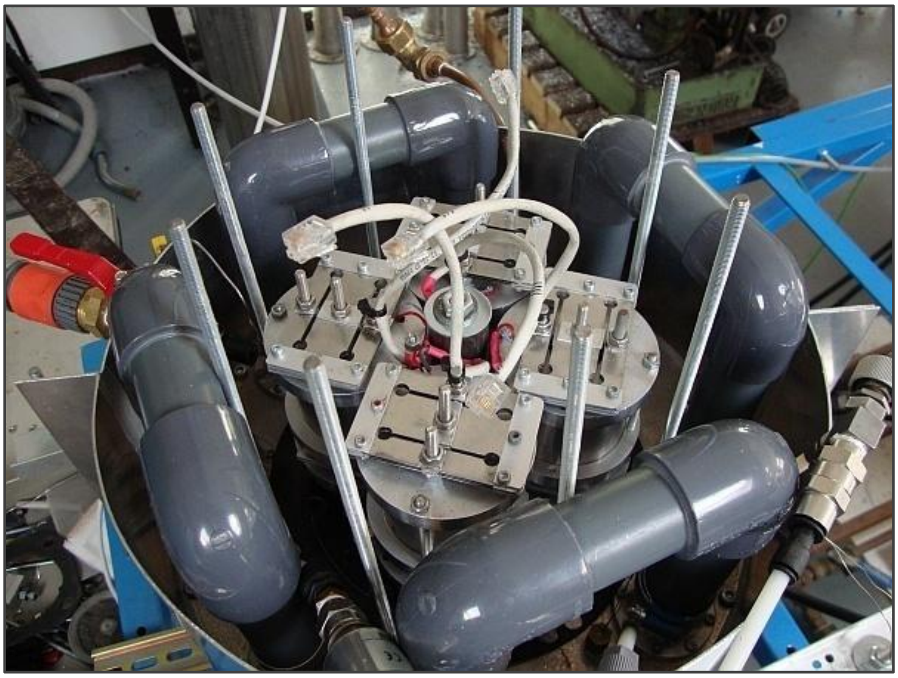


Figure 3.2: A thermo-acoustic generator with 4 linear alternators in the middle

Concluding, it can be said that an efficient linear alternator would be too heavy, too big and too expensive. Consequently, in this study the focus is on a bi-directional turbine which drives a rotating generator.

4. Analysis of bi-directional turbines

In this chapter the Wells turbine, the radial and axial impulse turbine are analyzed, mostly based on literature study. Furthermore, the pros and cons of these bi-directional turbines are considered. Prior to this, a function diagram is made and requirements are set which are given by Aster Thermakoestiche Systemen. The information gained in this chapter is an input for the trade-off study in chapter 5, where the best bi-directional turbine is chosen based on selection criteria. These selection criteria are chosen based on the requirements.

4.1. List of requirements and function diagram

In Figure 4.1 a function diagram of a bi-directional turbine is shown. One of the main functions of a bi-directional turbine is to convert a linear movement of air, which is the acoustic power, into a rotating movement. The rotational movement of the bi-directional turbine drives an electrical generator. Note that the bi-directional turbine turns always in the same direction regardless the direction of the airflow.

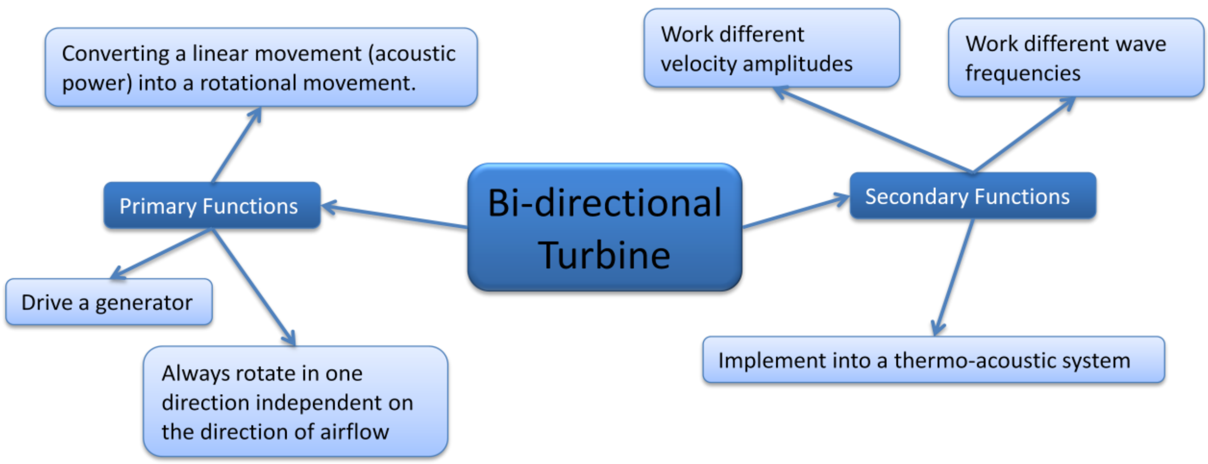


Figure 4.1: Function diagram of the turbine

Before analyzing the bi-directional turbines, requirements are defined. Based on these requirements the pros and cons of the bi-directional turbine types can be defined. Next, selection criteria are defined based on these requirements which are discussed in chapter 5.

Requirements

- High turbine efficiency at a wide range of input velocities;
- Simple design, which is easy to implement into the thermo-acoustic system;
- Low production costs of the bi-directional turbine;
- Good starting characteristics.

4.2. Wells turbine

The blades of a Wells turbine consist out of symmetrical airfoils. The plane of symmetry of the airfoils is perpendicular to the air stream. Figure 4.2 shows a schematic overview of the working principle of a Wells turbine.

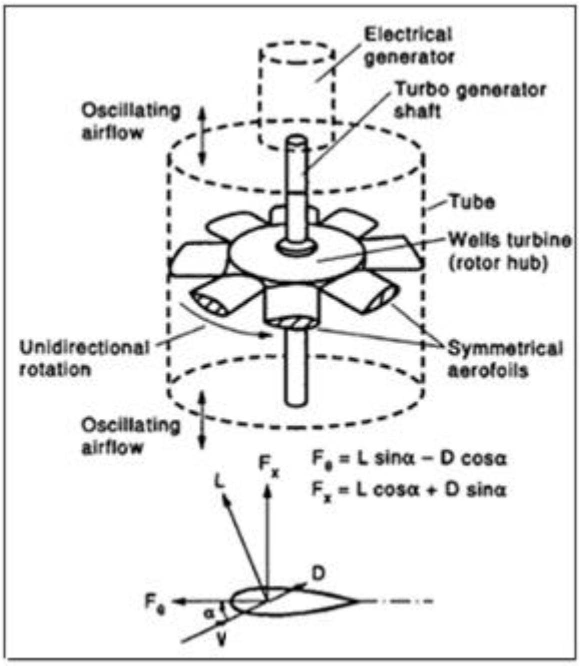


Figure 4.2: Schematic overview of the working principle of a Wells turbine

Referring to Figure 4.2 the force in axial and tangential direction can be calculated [6]:

$$F_x = L \cdot \cos\alpha + D \cdot \sin\alpha$$

$$F_\theta = L \cdot \sin\alpha - D \cdot \cos\alpha$$

The lift and draft force L and D can be calculated with the following formulas [1]:

$$L = \frac{1}{2} \cdot \rho \cdot {v_0}^2 \cdot S \cdot C_L$$

$$D = \frac{1}{2} \cdot \rho \cdot {v_0}^2 \cdot S \cdot C_D$$

Below, a simple calculation is made with these formulas. This is done to define the force in tangential and axial direction. The input values and results are shown in Table 4.1. The calculations are hand on a NACA0012S profile is used, which is a symmetrical airfoil [16].

Table 4.1: Input values and results for axial and tangential force of a Wells turbine

	Symbol	Value	Value	Value	Value	Value	Unit
Average velocity	v_0	2	6	10	14	18	m/s
Density	ρ	4.241	4.241	4.241	4.241	4.241	kg/m^3
Blade surface area	S	0.0003	0.0003	0.0003	0.0003	0.0003	m^2
Angle of attack	α	10	10	10	10	10	°
Lift coefficient	C_L	0.70	0.70	0.70	0.70	0.70	$-$
Drag coefficient	C_D	0.0275	0.0275	0.0275	0.0275	0.0275	$-$
Lift force	L	0.01	0.13	0.35	0.69	1.14	N
Drag force	D	0.00	0.00	0.01	0.03	0.04	N
Axial force	F_x	0.01	0.13	0.35	0.68	1.13	N
Tangential force	F_θ	0.00	0.02	0.05	0.09	0.15	N

Table 4.1 shows that the tangential force is much smaller at all velocity inputs. This is because a Wells turbine is a lift based concept, while a high tangential force is required. As a result of this the turbine efficiency is low.

Nevertheless, according to the configuration of the Wells turbine it is easy to implement into the thermo-acoustic system which can be seen in Figure 4.3. In this figure D_t is the tube diameter of the thermo-acoustic system. Besides, a Wells turbine is one of the most easiest and probably most economical bi-directional turbine, due to its simple geometry [10].

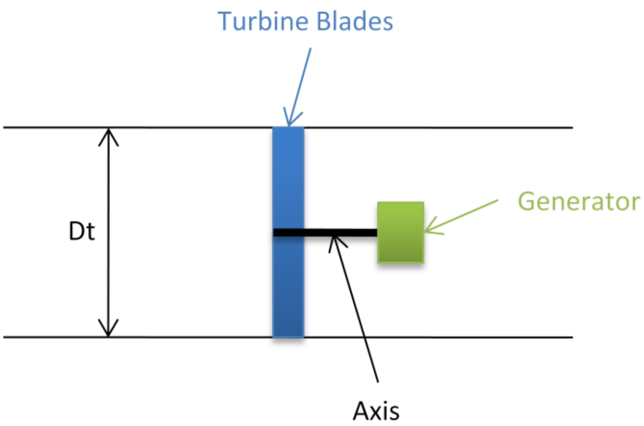


Figure 4.3: Schematic overview of the cinfiguration of a Wells turbine

On the other hand, the disadvantage of a Wells turbine is the poor starting characteristics [10]. A Wells turbine is not self-starting, unless guide vanes are used [3]. Guide vanes are fixed airfoils that direct the air into the turbine blades as shown in Figure 4.4.

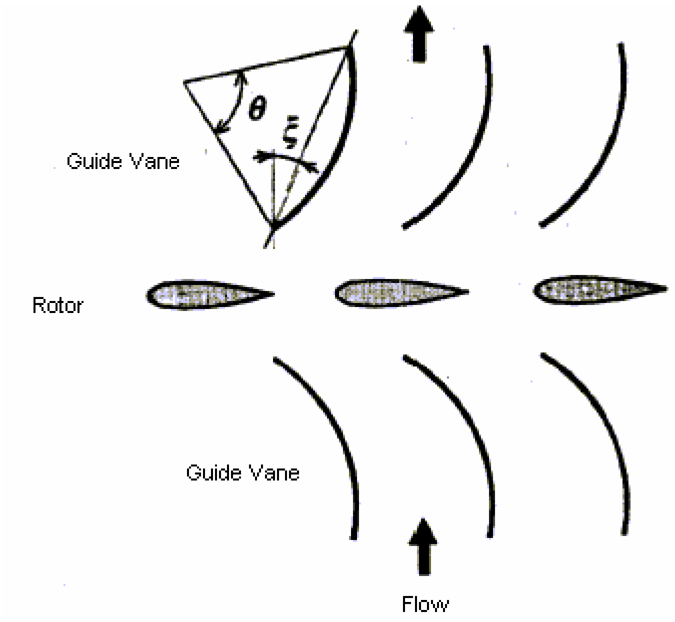


Figure 4.4: Wells turbine with Guide vanes

The acoustic power generated by the thermo-acoustic system is not constant, especially for the application of this system in developing countries. In other words, the average velocity v_0 changes constantly. Moreover, a high efficiency is only possible at high velocities [7]. The efficiency of a Wells turbine in a OWC is only 30% [10].

4.3. Impulse turbines

Alternatives of a Wells turbine in order to increase the turbine efficiency. One of these alternatives is an impulse turbine [7]. In this paragraph the general working principle of an impulse turbine is discussed, followed by two separate sub-paragraphs. In these sub-paragraphs the radial and axial impulse turbine are discussed.

In Figure 4.5 a schematic overview is given of the working principle of an impulse turbine is given. An impulse turbine consists out of three components. In the middle a rotating turbine is attached to a generator. At both sides fixed guide vanes are positioned. During operation the air flows into the guide vanes.

Guide vanes are fixed airfoils that direct the air into the moving turbine blades, increasing the turbine efficiency. After the passage through the guide vanes the air flows along the turbine blades. The flow of the air along the turbine blades results in an impulse, which is the green arrow in Figure 4.5. Finally, the air flows out of the turbine blades into the other fixed guide vanes. Thereafter, the same thing happens in opposite direction.

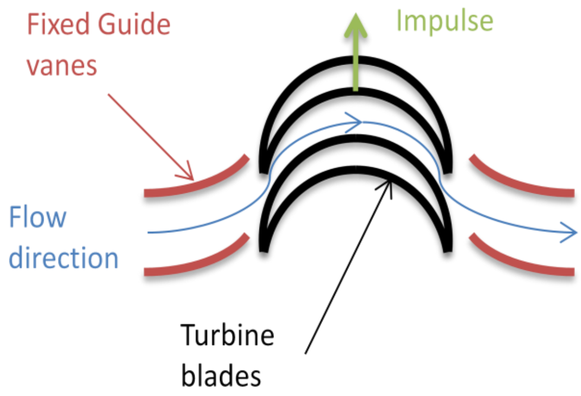


Figure 4.5: Working principle of an impulse turbine

Figure 4.6 shows a more detailed overview of the working principle of an impulse turbine, including velocity vectors. In this figure v_1 is the absolute velocity at the inlet (at the point where the fluid comes out of the guide vane), v_2 is the absolute velocity at the outlet (when the fluid comes out of the blades). Besides, v_{r_1} and v_{r_2} are respectively the relative velocities at the in- and outlet, which are perpendicular to β_1 (turbine blade inlet angle) and β_2 (turbine blade outlet angle). The guide vane inlet angle is given by the symbol α. This angle is smaller than blade angle β, due to this difference a third velocity vector is introduced, which is the blade velocity v_b.

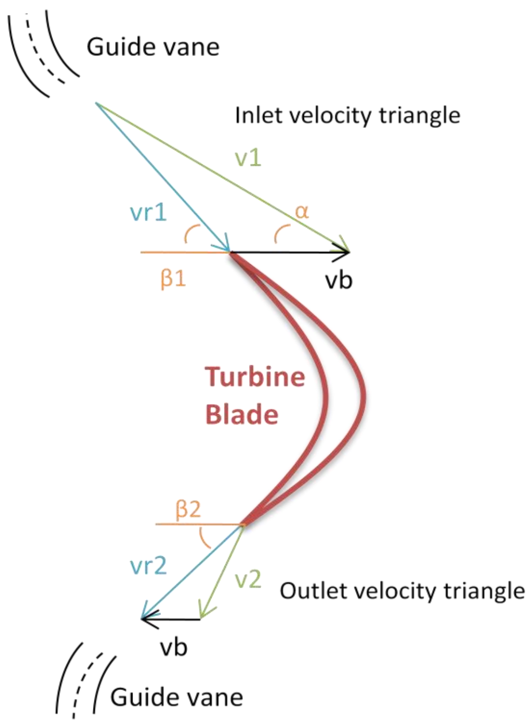


Figure 4.6: Overview working principle impulse turbine including velocity vectors

The efficiency of an axial impulse turbine depends on the guide vane inlet angle α [4]. The lower the inlet angle, the higher the efficiency of the turbine. Although there is a certain limit to this angle. This minimum angle of the guide vane is 12° [4]. When a lower angle is used the efficiency would increase theoretically, but in practice the efficiency is reduced. There is always a small gap between the guide vane section and the rotating turbine to prevent friction between these two components. At a very low angle of for instance $5°$ the air that flows out of the guide vanes flow next to instead of into the blade section which is shown in Figure 4.7. Therefore, the minimum guide vane angle that can be used is $12°$ [4].

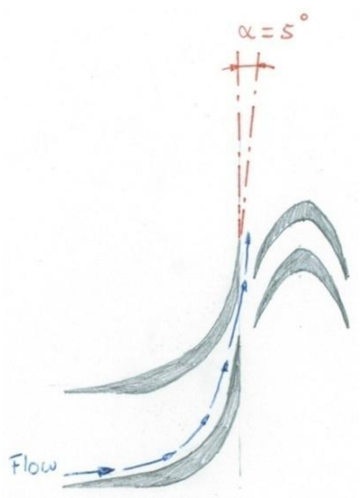


Figure 4.7: Guide vane with a low inlet angle (5°)

The reason for a low guide vane inlet angle can be explained with Figure 4.8, where α is the guide vane inlet angle and v_1 is the absolute velocity at the inlet. In this figure the left velocity triangle has a low guide vane inlet angle, while the right velocity triangle has a high guide vane inlet angle. This means that when a low guide vane inlet angle is used the velocity vector in tangential direction is bigger than when a higher guide vane inlet angle is used. A higher tangential velocity results in a higher impulse [4]. As mentioned earlier in paragraph 4.2 about Wells turbines the force in tangential direction, as a result of the tangential velocity, should be as high as possible compared to the axial force. This results in a higher turbine efficiency.

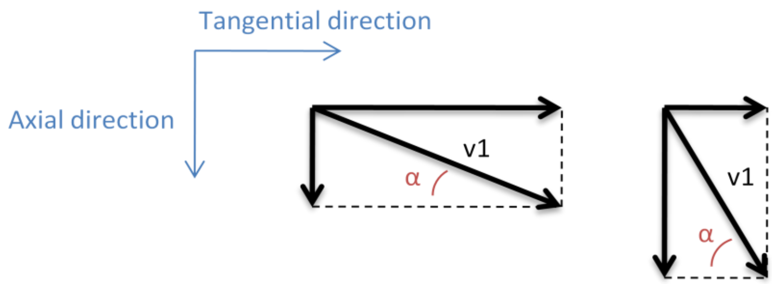


Figure 4.8: Effect of guide vane angle on efficiency

The efficiency of impulse turbines is ranges from $40-50\%$ [10], but under optimal conditions the efficiency of impulse turbines could even reach values up to 70% [7]. So, impulse turbines are more efficient than a Wells turbine. Another benefit of an impulse turbine is that high efficiencies could be reached under a wide range of input velocities, while a Wells turbine is only efficient at high velocity inputs [10]. Additionally, the starting characteristics of an impulse turbine are superior in comparison to a Wells turbine. Concluding, an impulse turbine seems more suitable for a thermo-acoustic system than a Wells turbine, also due to its wider operating ranges.

Now, the working principle of an impulse turbine is explained. In the upcoming two sub-paragraphs the difference between a radial and an axial impulse turbine are explained, including the pros and cons of both bi-directional turbine types.

4.3.1. Axial impulse turbines

Figure 4.9 shows an axial impulse turbine. When this turbine is implemented in the system, at both sides guide vanes are attached to guide the flow in the right direction. Side-views the guide vanes of an axial impulse turbine are shown schematically in Figure 4.10.

Figure 4.9: Axial impulse turbine

The benefit of an axial impulse turbine is that it is easy to implement into the thermo-acoustic system, because of its configuration which is in fact the same as the configuration of the Wells turbine. The efficiency of an impulse turbine is around 50% [10] and could reach to 70% as mentioned earlier in paragraph 4.3. On the other hand, the disadvantage of an axial impulse turbine compared to a Wells turbine is its geometry. The turbine blades are more complex than the symmetrical airfoils used in a Wells turbine. To design turbine blades for a Wells turbine standard dimensions of a symmetrical airfoil can be used, while the geometry of a blade of an impulse turbine depends on different angles, which are defined by efficiency calculations.

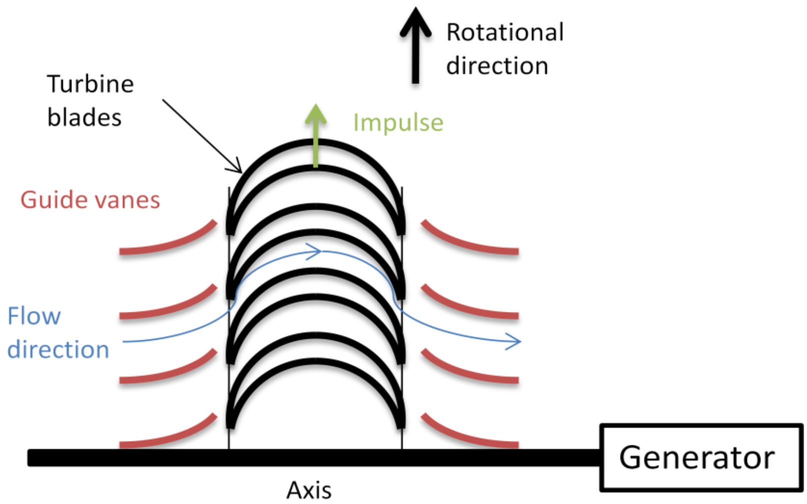


Figure 4.10: Side view of an axial impulse turbine

4.3.2. Radial impulse turbines

In a radial impulse the turbine blades and guide vanes are placed in a different way, which can be seen in Figure 4.11. A radial impulse turbine consists out of 2 components. A fixed component where the guide vanes are implemented, this is the left component shown in Figure 4.11. The other component is the turbine including the turbine blades, which is the right component in Figure 4.11.

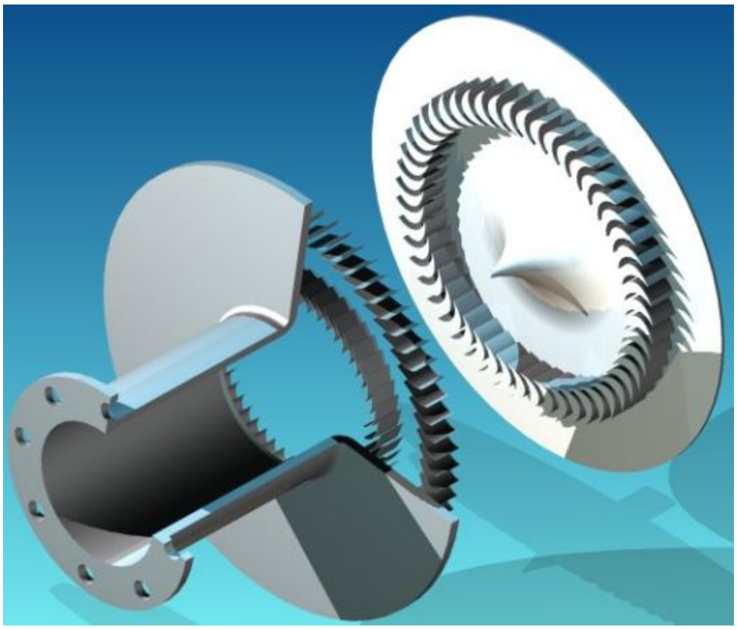


Figure 4.11: Detailed view of radial impulse turbine

In Figure 4.12 a schematical overview is given of several bi-directional turbine types. While a Wells turbine and an axial impulse turbine are axial flow turbines, a radial impulse turbine has a radial flow. During operation of a radial impulse turbine the air flows in at D_1 and leaves the turbine at the top at D_2, then the air flows in at D_2 and leaves at D_1. This movement is repeated constantly. D_1 and D_2 are shown at the left in Figure 4.12.

Compared to a Wells turbine and an axial impulse turbine a radial impulse turbine has a higher torque [7]. This because the radius of the rotating turbine in a radial impulse turbine is bigger, which results in a bigger arm. This can be derived from Figure 4.12. The effiency of a radial impulse is the same as the efficiency of an axial impulse turbine (50%) [7].

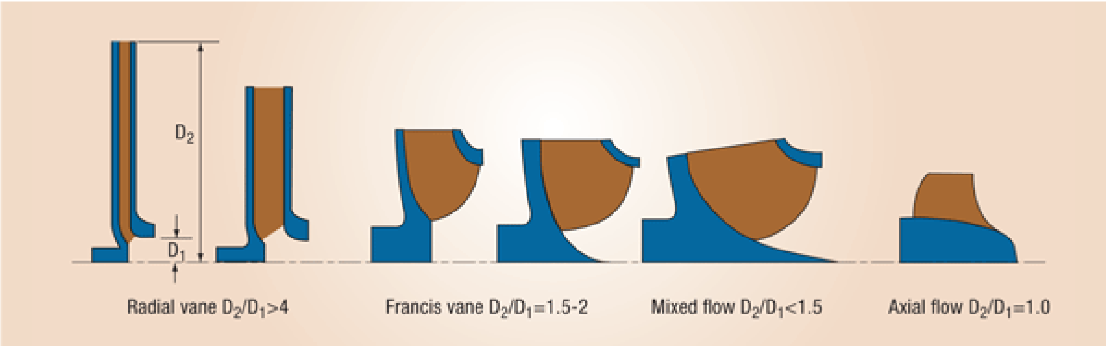


Figure 4.12: Bi-directional turbine types

On the other hand, according to Figure 4.12 D_2 is bigger than D_1 in a radial impulse turbine, which is not beneficial. This results in an output power of the generator which is not constant [7]. Another disadvantage of a radial is the complex design of the turbine blades, which can be seen in Figure 4.13. The inlet angle of the turbine blades, shown in the middle of Figure 4.13, are not the same at both sides ($19°$ and $36°$). These different angles are also a result of the not constant output power of the generator.

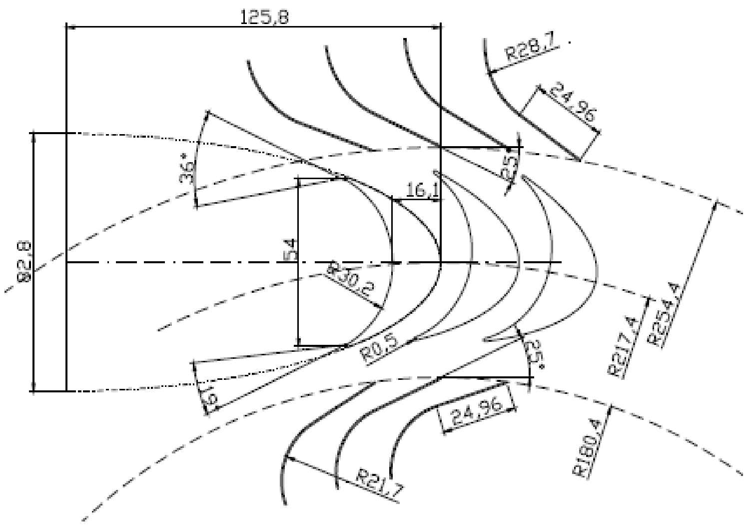


Figure 4.13: Geometry of a radial impulse turbine

Compared to the configuration of the Wells turbine and axial impulse turbine the radial impulse turbine it is more difficult to construct into the thermoacoustic system. This because at one side the air flows in in axial direction (D_1) and at the other side in tangential direction (D_2). Therefore, an extra chamber needs to be attached to the thermo-acoustic system which can be seen in Figure 4.14. This chamber is called the bouncing space, where the pressure is also $4\ bar$. The disadvantage is that the thermo-acoustic system becomes bigger, while a system as small as possible is required.

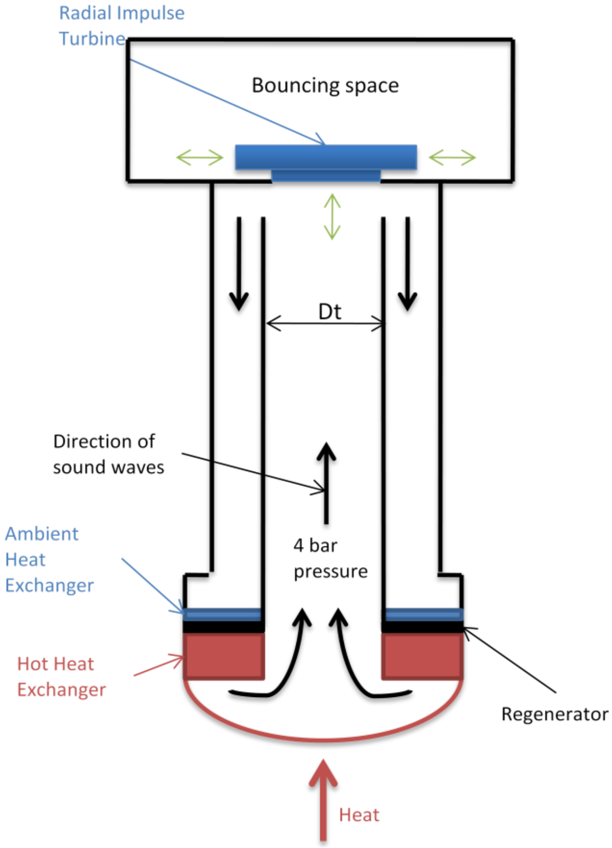


Figure 4.14: Schematic overview of a radial impulse turbine implemented in a thermo-acoustic system

4.4. Conslusion

The design of a Wells turbine is the simplest design, while the design of the radial impulse turbine is the most complex. Nevertheless, the axial and radial impulse turbine are the most efficient bi-directional turbine types, while the Wells turbine is less efficient. Besides the starting characteristics of the radial and axial impulse turbine are superior to the Wells turbine.

5. Trade-off study

In this chapter the bi-directional turbine is selected which is best suitable for the thermo-acoustic system. First, the selection criteria are defined followed by trade-off. Next, scores are given to every turbine type based on the pros and cons discussed in the previous chapter. Finally, the trade-off table is shown.

5.1. Selection criteria

In this sub-paragraph every selection criteria is described including the reason why it is used. A weight factor is given to every criteria with a scale of 1-3. Weight factors are used because some criteria are more important than the others.

5.1.1. Efficiency

One of the sub-questions of the project is: What is the efficiency of the final design of the turbine? This criterion requires also high efficiencies of the bi-directional turbine at a wide range of input velocities. Because the turbine efficiency is of high importance in this project, it gets the highest weight factor of 3.

5.1.2. Starting characteristics

As mentioned earlier in chapter 4 good starting characteristics of the bi-directional turbine are required. Because the input velocity of the turbine changes continuously, the turbine has to respond quickly to these changes, especially when the input velocity increases. Bi-directional turbines become more efficient at higher velocity inputs [10]. Therefore, a bi-directional turbine is needed with good starting characteristics. So, this criterion gets a weight factor of 2.

5.1.3. Configuration

This criterion is about the configuration of the bi-directional turbine, which has an effect on the construction into the thermo-acoustic system. According to a requirement, mentioned in 4.1, the design of the bi-directional turbine should be easy to implement into the thermo-acoustic system. When the turbine is easy to implement into the thermo-acoustic system, the thermo-acoustic system becomes smaller but also cheaper. For the application of the thermo-acoustic system in developing countries, the total price of the system should be as low as possible. Thus, this criterion gets a weight factor of 2.

5.1.4. Complexity

The complexity of the design of the bi-directional turbine gets a weight factor of 1. This criteria is focused on the manufacturing and assembly of the design of the bi-directional turbine. Simple construction means lower price of the system. Nevertheless, in the future mass production is done with injection molding, which makes production very cheap regardless the complexity of the design. The turbine is made out of one part, which gives a lot of freedom in the design.

5.2. Scores

The selection criteria and the pros and cons of every turbine type are known the bi-directional turbine type can be chosen which is best applicable for the designed thermo-acoustic system, where a scale of 1-5 is used. In this paragraph the scores of every bi-directional turbine are given, including motivations.

5.2.1. Efficiency

In Table 5.1 an overview is shown of the efficiency of every turbine type. Because the turbine efficiency essential to the turbine choice, the turbine with the highest efficiency gets the highest score of 5 which is the axial impulse turbine. Besides the highest efficiency, the axial impulse turbine has high efficiencies over a wide range of input velocities. The radial impulse is also an efficient turbine with 50%, but it has some disadvantages compared to the axial impulse turbine. Because of the geometry, explained in 4.3.2, the efficiency is less constant than the efficiency of an axial impulse turbine. The radial impulse turbine has different efficiency values in both directions, because the guide vane inlet angles are different at both sides which can be seen in Figure 4.13. A turbine is needed with an efficiency which is as constant as possible. Therefore, the radial impulse turbine gets a score of 4. Finally, the efficiency of a Wells turbine is the lowest of all three turbines, high efficiencies are reached only at high velocity inputs. Due to this, the Wells turbine is not applicable for a thermo-acoustic systems. So, the Wells turbine gets a score of 1.

Table 5.1: Turbine efficiencies

	Efficiency
Wells turbine	30%
Axial impulse turbine	50%
Radial impulse turbine	50%

5.2.2. Starting characteristics

The starting characteristics of a impulse turbines are superior compared to the starting characteristics of a Wells turbine [10]. More specific, for a Wells turbine it takes 2 to 3 times longer until the turbine rotates efficiently than an impulse turbine [10]. Therefore, the Wells turbine gets a score of 1. Because the axial an radial impulse turbine have much better starting characteristics, both get a score of 4, but not the highest score of 5. This is done because there is another improvement of impulse turbines which improves the starting characteristics even more. This is the use of self-pitch-controlled guide vanes, shown in Figure 5.1. Nevertheless, the use of self-pitch-controlled guide vanes increases the maintenance costs. Furthermore, the bi-directional turbine in the thermo-acoustic system is very small, maximum diameter of $84.6mm$, which makes the use of self-pitch-controlled guide vanes too complex.

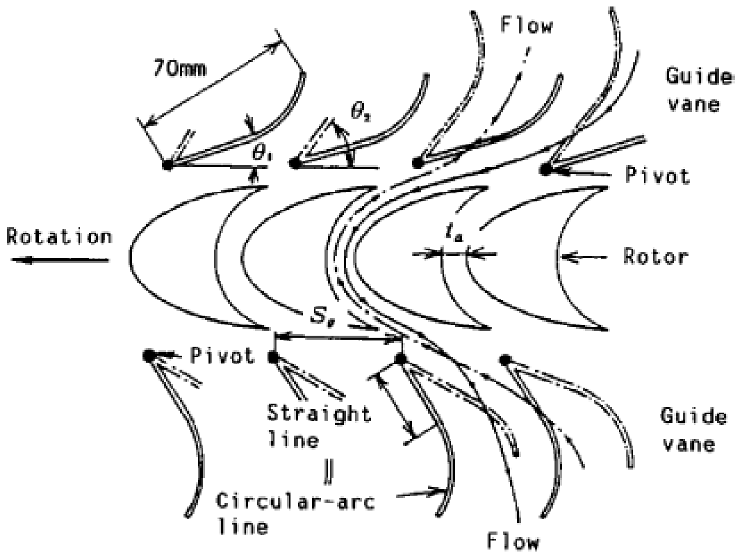


Figure 5.1: Impulse turbine with self-pitch-controlled guide vanes

5.2.3. Configuration

The configuration of the Wells turbine and axial impulse turbine are the same, because both are axial flow turbines. Compared to every other possible configuration the Wells turbine and axial impulse turbine are superior, because these bi-directional turbines can be implemented into the thermo-acoustic system very easily as shown in Figure 2.1 and Figure 4.3. Therefore, the Wells turbine and the axial impulse turbine get the highest score of 5, while the radial impulse turbine gets a score of 2. As explained in 4.3.2 an extra bouncing space need to be added to the thermo-acoustic system to be able to implement the radial impulse turbine into the system. This results in a bigger and heavier thermo-acoustic system. On the other hand, the thermo-system becomes not way bigger. Therefore, the score is 2 instead of 1.

5.2.4. Complexity

The Wells turbine gets the highest score of 5, due to its simple geometry [10] caused by the simple symmetrical airfoils. Meanwhile, the blades of the axial impulse turbine are more complex than the airfoils of the Wells turbine. On the other hand, the turbine blades are still symmetrical, therefore the score of the axial impulse turbine is 3. The turbine blades of the radial impulse turbine have the most complex form of all 3 turbine types, which can be seen in Figure 4.13. Therefore, it costs more time to design a radial impulse turbine, therefore the radial impulse turbine gets the lowest score of 1.

5.3. Conclusion

In Table 5.2 the trade-off table is shown. Concluding, an axial impulse turbine is by far the best applicable bi-directional turbine for a thermo-acoustic system. On all criteria the axial impulse turbine has high scores. The only disadvantage is that the design is complex, on the other hand it is the most efficient turbine, which is most important. Although a Wells turbine has an easy design and it is easy to construct into a thermo-acoustic system, it is not efficient enough and not self-starting. Next to the axial impulse turbine the radial impulse turbine is efficient too, including good starting characteristics. However, the design is too complex and to implement a radial impulse turbine into a thermo-acoustic system the thermo-acoustic system becomes bigger which is not beneficial.

Table 5.2: trade-off table

		Wells turbine		**Axial impulse turbine**		**Radial impulse turbine**	
	Weight factor	Score	**Total score**	Score	**Total score**	Score	**Total score**
Efficiency	*3*	1	**3**	5	**15**	5	**15**
Starting characteristics	*2*	1	**2**	4	**8**	4	**8**
Configuration	*2*	5	**10**	5	**10**	2	**4**
Complexity	*1*	5	**5**	3	**3**	1	**1**
Total			**20**		**36**		**28**

6. Final design

For the final design an axial impulse turbine is chosen. In this chapter the parameters and the exact configuration of the design are defined. In Figure 6.1 the final design of the axial impulse turbine is shown. The configuration of the axial impulse turbine is shown in Figure 6.2.

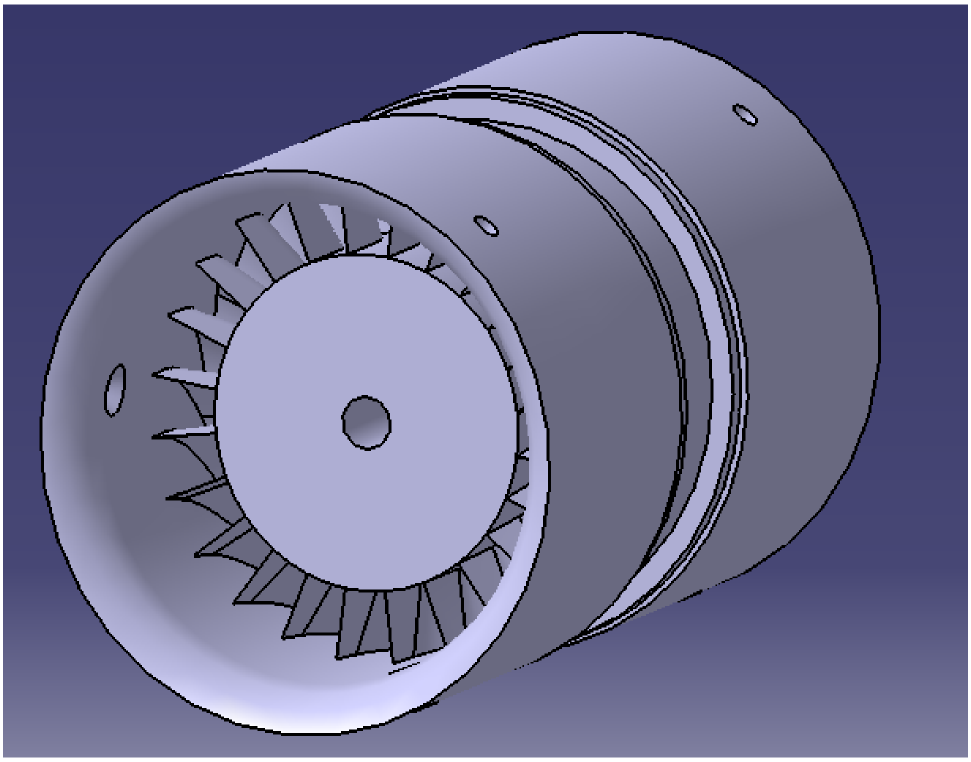


Figure 6.1: Final design of axial impulse turbine

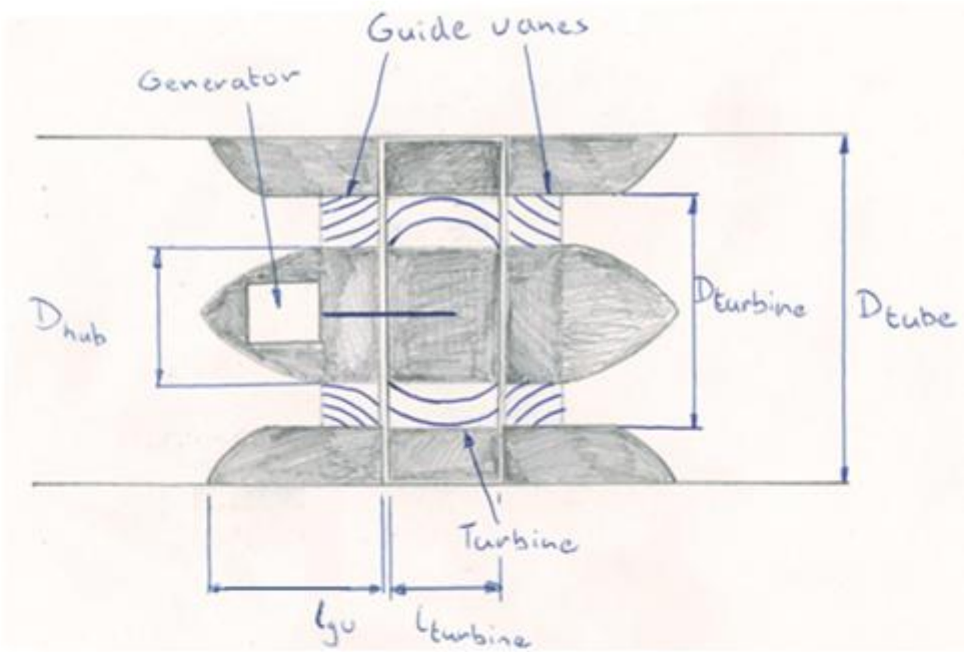


Figure 6.2: Configuration of the final design

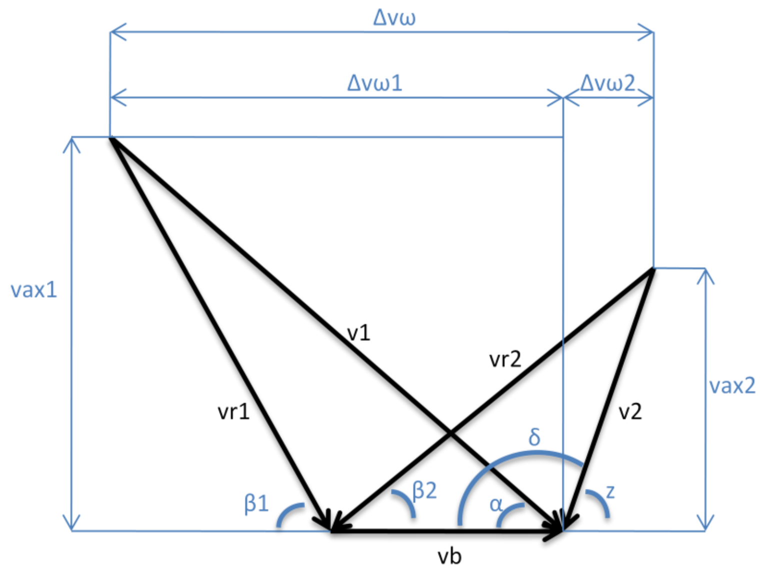
All important parameters to calculate the final design of the axial impulse turbine are shown in Table 6.1. In the rest of this chapter all these parameters are explained separately, including calculations. Subsequently, the turbine efficiency and the turbine and generator work of the final design of the axial impulse turbine are defined. The work is the amount of energy generated by an object (in this project the bi-directional turbine or generator) and is expressed in Watt.

Table 6.1: Parameters final design of axial impulse turbine

Parameter	Symbol	Value	Unit
Tube diameter	D_{tube}	84.6	mm
Turbine diameter	$D_{turbine}$	72	mm
Turbine hub diameter	D_{hub}	50.4	mm
Guide vane section length	l_{gv}	54.29	mm
Turbine length	$l_{turbine}$	25	mm
Total length	l_{total}	135.58	mm
Guide vane angle	α	12	°
Blade inlet angle	β	23	°

6.1. Turbine efficiency and output power

To calculate the efficiency and the output power of an axial impulse turbine, first a velocity diagram is needed [4]. The velocity diagram is shown in Figure 6.3, which is derived from Figure 4.6. In this figure v_1 is the absolute velocity at the inlet (at the point where the fluid comes out of the guide vane), v_2 is the absolute velocity at the outlet (when the fluid comes out of the blades). Besides, v_{r_1} and v_{r_2} are respectively the relative velocity at the in- and outlet, which are perpendicular to β_1 (blade inlet angle) and β_2 (blade outlet angle). The guide vane inlet angle is given by the symbol α. This angle is smaller than blade angle β, due to this difference a third velocity vector is introduced, which is the blade velocity v_b.



Index:
Δvω1= Tangential inlet velocity
Δvω2 = Tangential outlet velocity
vax1 = Axial inlet velocity
vax2= Axial outlet velocity
v1 = Absolute inlet velocity
v2 = Absolute outlet velocity
vr1 = Relative inlet velocity
vr2 = Relative outlet velocity
α = Guide vane inlet angle
β1 = Blade inlet angle
β2 = Blade outlet angle

Figure 6.3: Velocity diagram of an impulse turbine

The absolute velocity v_1 is one of the two input values to calculate the turbine efficiency [4]. In other words, v_1 can also be seen as the input velocity which is calculated in paragraph 2.2 ($9.96\,m/s$). The other input value for the turbine efficiency is the guide vane inlet angle α. To calculate the turbine efficiency and output power of the axial impulse turbine the following calculation method is used [4]:

First, the cross-sectional areas in the tube and in the turbine are calculated. The cross-sectional area in the turbine is smaller than in the tube, which can be seen in Figure 6.2. This is done in order to increase the input velocity. With a higher input velocity the output power of the axial impulse turbine increases with a factor of v^3 [4]. Therefore, the following two formulas are used:

$$A_{tube} = \left(\frac{\pi}{4} \cdot D_{tube}^2\right)$$

$$A_{turbine} = \left(\frac{\pi}{4} \cdot D_{turbine}^2\right) - \left(\frac{\pi}{4} \cdot D_{hub}^2\right)$$

Now, the input velocity or the absolute velocity v_1 can be calculated [1]:

$$v_1 = \frac{A_{tube}}{A_{turbine}} \cdot v_0$$

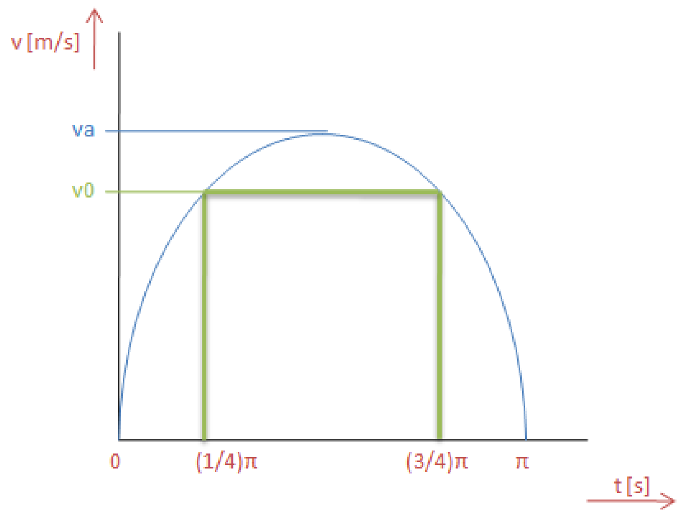


Figure 6.4: Input velocity at a half period of the sound wave (sinefunction)

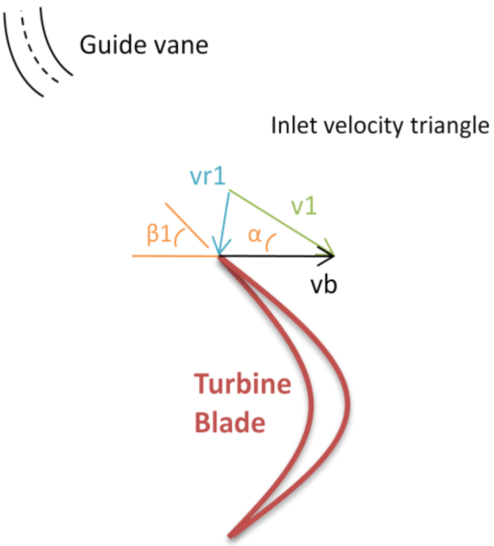
The efficiency of the turbine is dependent on the guide vane inlet angle α. For a most efficient turbine an angle of $12°$ [4] is used, which results in an efficiency of 0.94. So, to calculate the turbine efficiency the following formula is used [4]:

$$\eta_D = \left[\frac{\cos^2 \alpha}{2} \cdot (1 + k_b)\right]$$

Where k_b is the blade friction factor. Due to the friction of the air flow along the blades energy is lost. For turbine blades with a smooth surface a blade friction factor of 1 can be assumed, which means that there are no losses of energy. The surface of the blades of the designed axial impulse turbine are smooth too. Nevertheless, a blade friction factor of 0.97 is assumed in the calculations.

Even though the loss of energy is small, there is always a loss The turbine efficiency calculated with the above mentioned formula is at a constant velocity. Figure 6.4 shows the varying input velocity during half a period, where v_0 is the average velocity, v_1 is the velocity amplitude. In operation the turbine is working efficiently between $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$. So, only during a half of this half period of the sound wave [5]. Figure 6.5 shows the velocity triangle of the axial impulse turbine at the inlet at a low input velocity (lower than v_0). During this situation the direction of the velocity (v_1) is the same, because of the guide vane angle α. The blade velocity is the same as during the optimal condition shown in Figure 4.6, while the direction of the relative velocity changes as shown in Figure 6.5. The air is flowing against the side of the turbine blades. This reduces the rotational speed of the turbine. Therefore, the a calculation factor of 0.5 is chosen as shown in the formula below, which results in a turbine efficiency of 0.47 [4]:

$$\eta_D = 0.5 \cdot \left[\frac{\cos^2 \alpha}{2} \cdot (1 + k_b)\right]$$



Index:
v1 = Absolute inlet velocity
vr1 = Relative inlet velocity
vb = Blade velocity
α = Guide vane inlet angle
β1 = Blade inlet angle

Figure 6.5: Velocity triangle al low input velocity

Subsequently, the blade velocity v_b and the blade inlet angle β can be defined [4]:

$$v_b = v_1 \cdot \frac{\cos \alpha}{2}$$

$$\tan \beta_1 = \frac{v_{ax_1}}{\Delta v_{\omega_1}}$$

Where,

$$v_{ax_1} = v_1 \cdot \sin \alpha$$

and

$$\Delta v_{\omega_1} = v_1 \cdot \cos \alpha - v_b$$

Now, the tangential thrust can be calculated. The tangential thrust is needed to calculated the turbine work [4].

$$P_t = \dot{m} \cdot \Delta v_\omega$$

Where $\dot{m}$ is the mass flow and

$$\Delta v_\omega = \frac{\eta_D \cdot v_1^2}{2 \cdot v_b}$$

Finally, the rotational speed and the turbine and generator work can be calculated with the two following formulas [4]:

$$n = \left(\frac{v_b}{crc}\right) \cdot 60$$

$$W_D = P_t \cdot v_b$$

$$W_G = 0.8 \cdot W_D$$

Where W_D and W_G are the turbine and generator work, n is the rotational speed in rpm and crc is the circumference of the turbine. The generator work has an efficiency of 80% [C. de Blok, personal communication, 18 April 2012]. Above, only the important formulas are shown. A detailed calculation can be found in Appendix B. Now, the efficiency, turbine work and the generator work can be calculated. In Table 6.2 the input value are shown the calculate these values. The results are shown in Table 6.3.

Table 6.2: Input values to calculate the turbine efficiency, turbine work and generator work

Parameter	Symbol	Value	Unit
Tube diameter	D_{tube}	84.6	mm
Turbine diameter	$D_{turbine}$	72	mm
Turbine hub diameter	D_{hub}	50.4	mm
Input velocity	v_0	9.96	m/s
Guide vane inlet angle	α	12	°
Blade friction factor	k_b	0.97	-
Mass flow	$\dot{m}$	0.237	kg/s
Turbine circumference	crc	0.23	m

Table 6.3: Turbine efficiency, turbine work and generator work

Parameter	Symbol	Value	Unit
Turbine efficiency	η	0.47	-
Turbine work	W_D	40.7	W
Generator work	W_G	32.5	W
Rotational speed	n	3441	rpm

The results shown in Table 6.3 show that an output power of 50 Watt of electricity (generator work) is not possible. In chapter 0 these results are compared with the test results.

6.2. Turbine diameter

The turbine diameter has a direct effect on the power output of the thermo-acoustic system, because the smaller the turbine diameter compared to the diameter of the tube the higher the velocity v_1. Finally, a turbine diameter of $72mm$ is chosen which can be seen in Table 6.2. This is defined so that the turbine fits into the tube of the thermo-acoustic system.

6.3. Hub-tip ratio

The hub-tip ratio is the ratio between the inner and outer diameter of the turbine blades. In the left image of Figure 6.6 the hub of the turbine blade is the black circle in the middle. The hub-tip ratio is important to the performance of the turbine. When the hub is bigger, the blade section is smaller which increases the air velocity. For an optimal turbine efficiency hub-tip ratio a value between 0.6 and 0.7 is needed [13]. A hub-tip ratio 0.7 is chosen in order to increase the air velocity are much as possible. Now, the diameter of the hub can be calculated:

$$D_{hub} = 0.7 \cdot D_{turbine}$$

$$D_{hub} = 0.7 \cdot 72$$

$$D_{hub} = 50.4mm$$

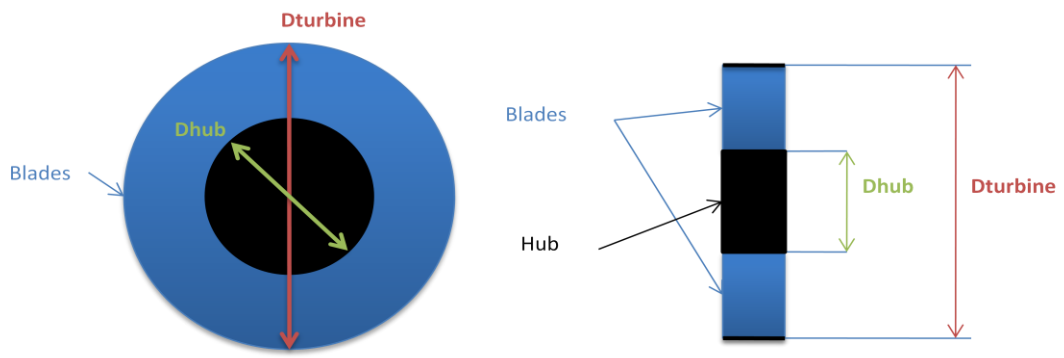


Figure 6.6: Hub-tip ratio, front-view (left), side view (right)

6.4. Guide vanes

As mentioned earlier in 6.1, for an optimal turbine efficiency a guide vane inlet angle in 12° is used [4]. In this paragraph the other dimensions of the guide vane are discussed.

In Figure 6.7 the dimensions of a single guide vane are given. In this figure can be seen that the length of the guide vanes is $25mm$. The goal is to design a bi-directional turbine which is as small as possible. Therefore, the shortest possible length of the guide vane is chosen which is $25mm$. When a smaller length is chosen the guide vane inlet angle of 12° cannot be reached. In other words, the guide vane design does not fit anymore.

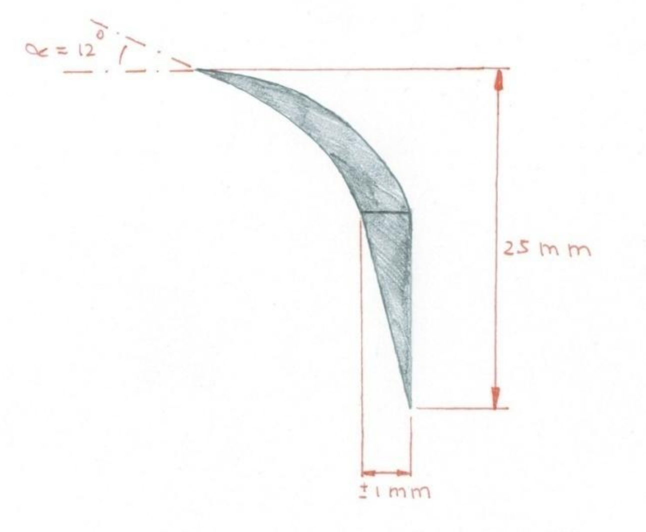


Figure 6.7: Dimensions of a guide vane

Finally, the maximum width of a guide vane should be at least around $1mm$. When a smaller width is used it is not possible to produce the guide vanes with 3D print techniques. Therefore, a minimum width of $\pm 1mm$ is chosen to be able to increase the number of guide vanes as much as possible. This resulted in a total number of guide vanes is 24. When more guide vanes are used the guide vanes would touch each other.

In Figure 6.8 a CATIA drawing is shown of the final design of the guide vane section. This figure shows that there are 3 small holes in the design. These are used to attach the guide vane section to the tube of the thermo-acoustic system.

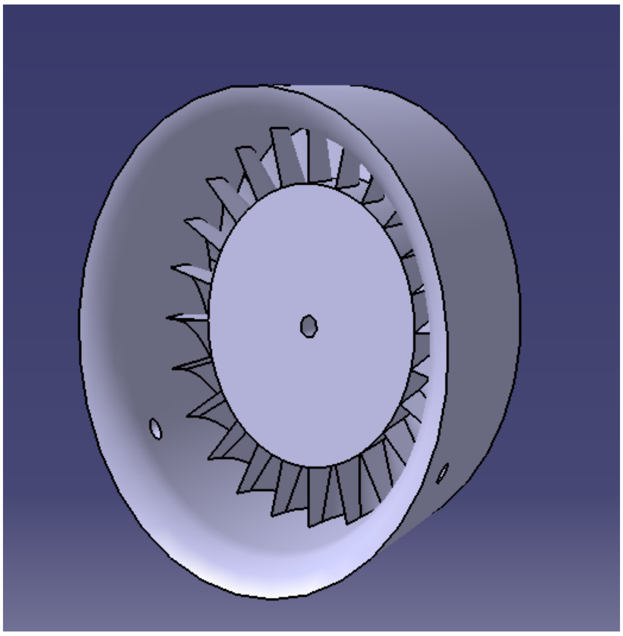


Figure 6.8: CATIA drawing of the guide vane section

6.5. Turbine blades

The blade inlet angle for the turbine blades can be defined based on the guide vane angle. Therefore, the following formula is used:

$$\beta = \tan^{-1}\left(\frac{v_1 \cdot \sin\alpha}{v_1 \cdot \cos\alpha - v_b}\right)$$

Where v_1 is the absolute velocity, v_b is the blade velocity and α is the guide vane inlet angle. The values for these inputs are shown in Table 6.4.

Table 6.4: Input values to calculate the blade inlet angle

Parameter	Symbol	Value	Unit
Absolute velocity	v_1	26.96	m/s
Blade velocity	v_b	13.19	m/s
Guide vane inlet angle	α	12	°

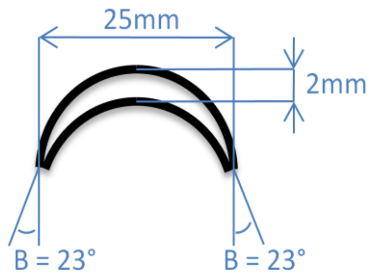


Figure 6.9: Dimensions of a turbine blade

This results in a blade inlet angle of 23°, which is an input to define the other dimensions of the turbine blades. An overview of the dimensions of a turbine blade are shown in Figure 6.9. The length of the blade section is the same as the length of the guide vanes, which is $25mm$ too. This is the shortest possible length. When a smaller length is chosen, the guide vane inlet angle of 23° is not possible. The minimum length of $25mm$ is chosen to make the axial impulse turbine as small as possible. Furthermore, 30 turbine blades are used. This is the maximum possible number of blades that could be implemented in the turbine. The higher the number of turbine blades, the higher the torque of the turbine [10]. A high torque is needed to improve the output power of the generator. When he turbine is rotating it gets a counteracting force of the generator. This reduces the rotational speed of the turbine, which results in a lower output power of the generator. In Figure 6.10 a CATIA drawing is shown of the final design of the turbine.

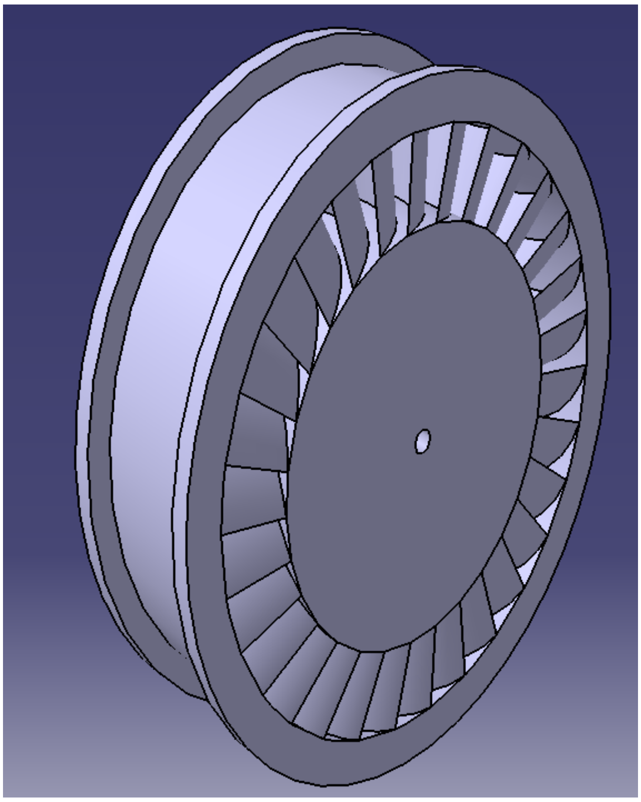


Figure 6.10: CATIA drawing of the turbine

6.6. Wave frequency

As mentioned earlier in 2.3 the effect of the frequency on the turbine efficiency is analyzed. One can assume that a particle of air has to pass through the whole turbine during a half period for a maximum efficiency. When a higher frequency is used the thermo-acoustic system becomes smaller. Therefore, an frequency is chosen with the following calculation:

$$T_{1/2} = \frac{1}{2} \cdot \frac{1}{f}$$

Where f is the wave frequency in Hz and $T_{1/2}$ is a half period. Besides, the average velocity through the whole turbine is $19.36\, m/s$. The total length of the turbine is $135.58mm$. this means that at an optimal wave frequency a particle of air has to pass this length during a half period.

A graph is shown in Figure 6.11 where $s_{1/2}$ is the travelled distance. According to this graph the wave frequency for an optimal efficiency is $70Hz$. In chapter 0 this result is compared with the test results to find out if this assumption is true.

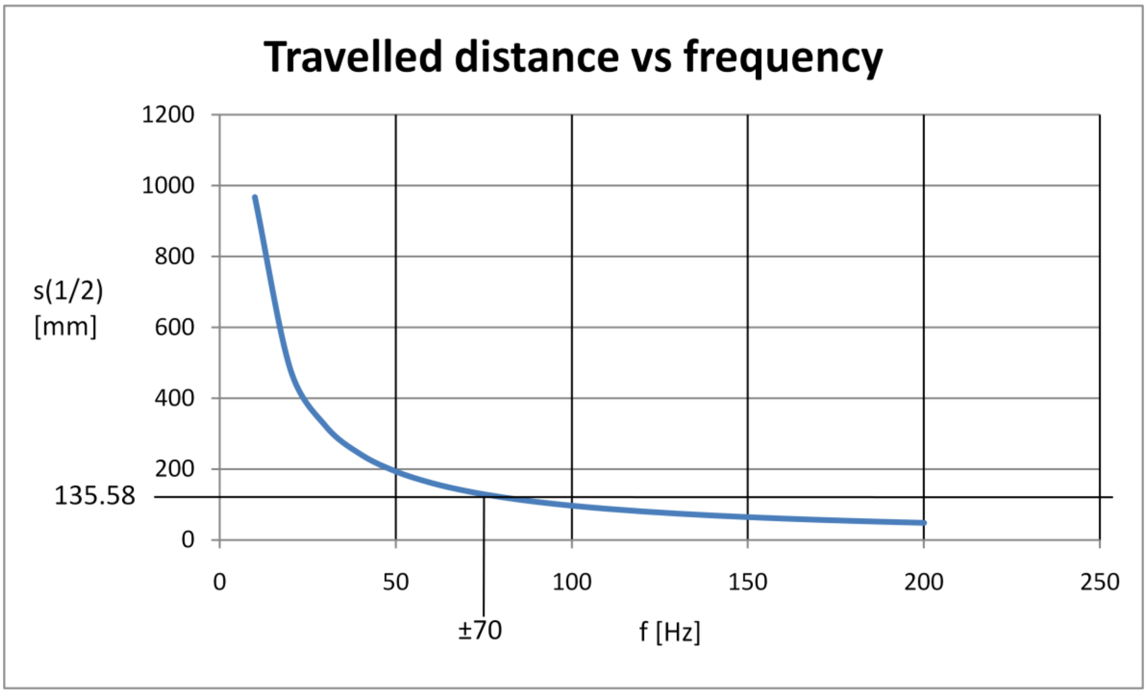


Figure 6.11: Travelled distance of a particle of air at different wave frequencies

6.7. Generator (brushless outrunner)

The generator which is attached to the turbine generates the electricity. The type of generator which is used is called a brushless outrunner. It consists out of an inner and an outer part which are shown in Figure 6.12. The inner part is fixed and has coils. Meanwhile, the outer part is rotating and it has magnets. Besides, the outer part is attached to an axis. This axis is attached to the turbine.

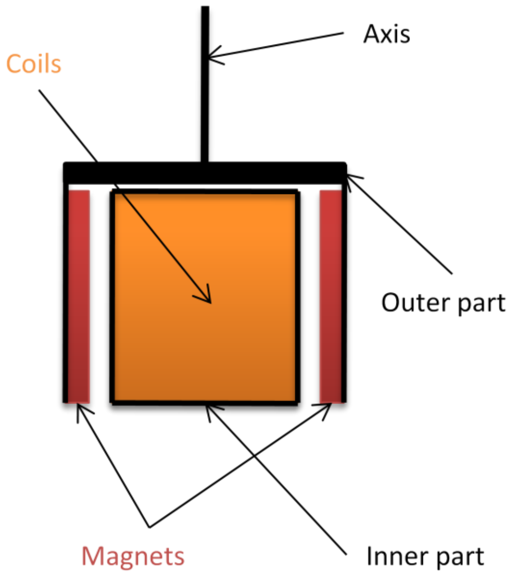


Figure 6.12: Schematic overview of a brushless outrunner

After research it is found out that the best applicable brushless outrunners are from the brand Hacker. Below, the chosen outrunner is discussed:

Hacker A10-13L

For the tests a Hacker A10-13L is used which has a diameter of $21mm$ and a length of $25mm$, it has a power range with a maximum of $75W$. Besides, the rotational speed is $1200RPM/V$. The unit RPM/V is for rounds per minute per voltage. In Figure 6.13 a picture of the Hacker A10-13L is shown.

Figure 6.13: Hacker A10-13L

6.8. Production of the final design

The production of the final design is done by an external company called Rapid Prototyping, which is a department of the company JB Ventures BV from Nijverdal. It is a small company which is specialized in 3D-printing of mainly prototypes or small series of around 200 products. 3D-printing is a new production technique which is mainly suitable for prototypes.

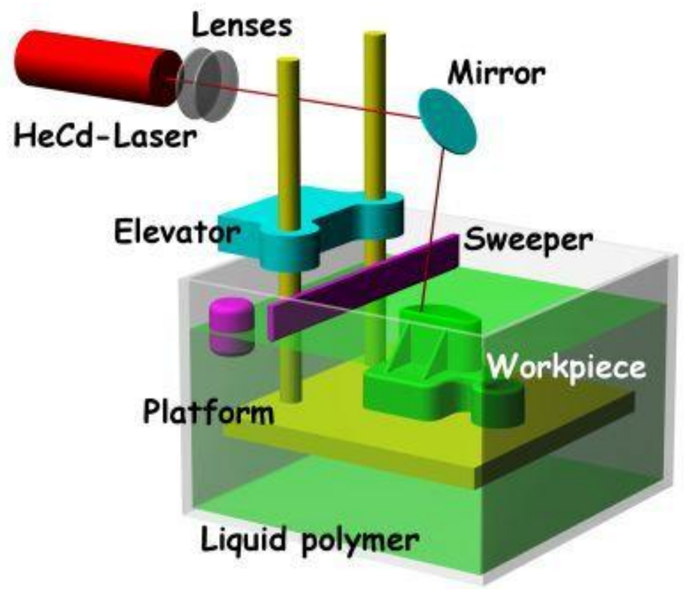


Figure 6.14: Stereolithography

JB Ventures BV produces the 3 components of the final design, which are the two guide vane sections and the turbine itself. The components are made by using stereolithography (SLA). With this production method it is possible to make products with a very smooth surface, which results in a high performance of the turbine. Stereolithography works in the following way: liquid is placed in a basin. A platform is placed in the basin which can be seen in Figure 6.14. The platform is placed near the upper surface of the fluid. By using a laser a part of the fluid is cured. Then the platform drops a bit and the next layer of the product is cured, so the product is built up out of small layers [11]. An overview of the production process of stereolithography is given in Figure 6.14. In Figure 6.15 a picture of one of the guide vane sections is shown. A photo of the turbine section is shown in Figure 6.16.

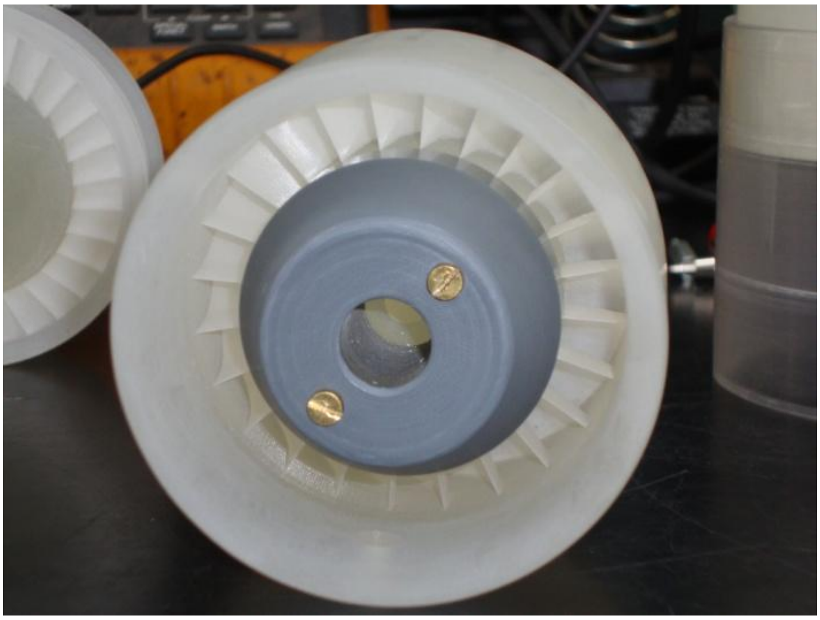


Figure 6.15: Guide vane section

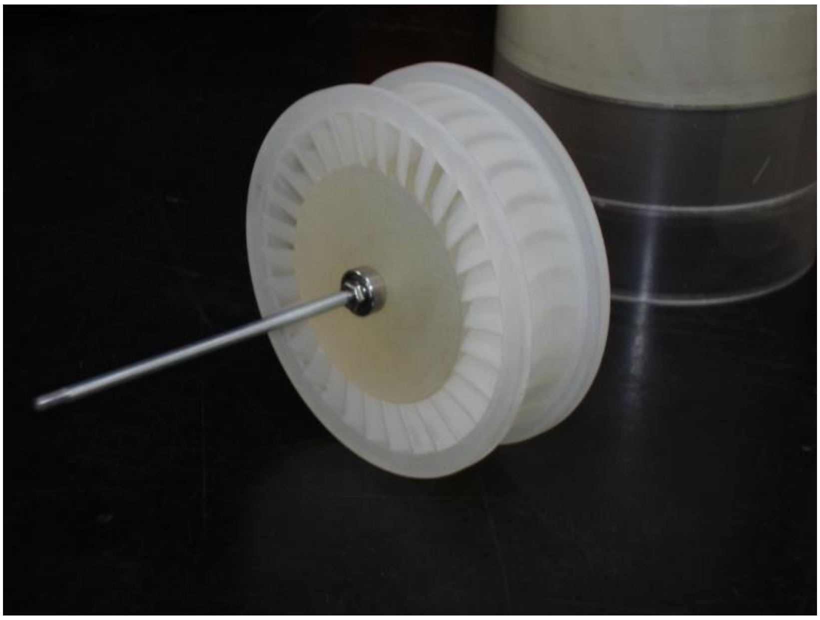


Figure 6.16: Turbine section

6.9. Conclusion

Now, all parameters of the detailed design of the axial impulse turbine are defined. Theoretically, the axial impulse turbine has an efficiency of 47% with a rotational speed of $3441\ rpm$ and an output power of $40.7W$. With a generator efficiency of 80% the generator should be able to produce $32.5W$ of electricity. In the next chapter these results are compared with the test results.

7. Test analysis

The main goal of this chapter is to compare the test results with the theoretical calculations made in the previous chapter. With this analysis the effect of the wave frequency on the turbine efficiency can be determined. Next, the turbine work and efficiency in practice can be defined. Finally, the following question can be answered: Is it possible to reach an output power of 50 Watt of electricity?

7.1. Test set-up

The turbine tests are performed under different conditions, because the thermo-acoustic system designed by Aster is not finished. Therefore, an acoustic impedance meter is used. An acoustic impedance meter generates a sound wave and measures the acoustic power that goes into the turbine. In Figure 7.1 the test set-up is shown with the turbine connected to the acoustic impedance meter. Besides, in Figure 7.2 a schematic overview is given of this acoustic impedance meter.

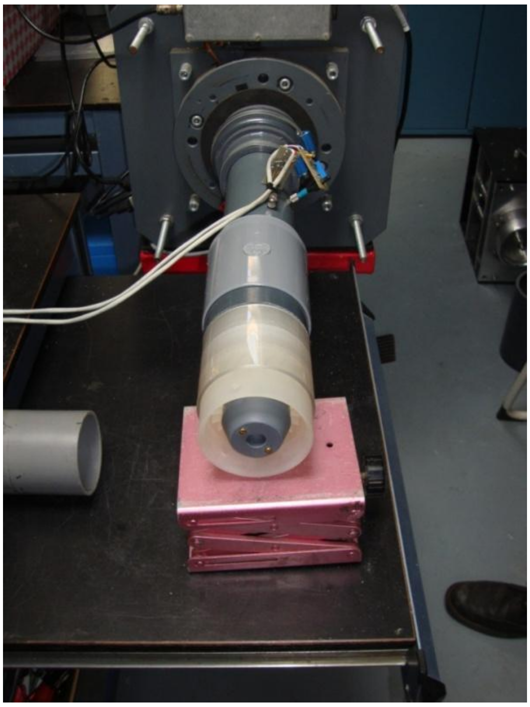


Figure 7.1: Test set-up of the prototype of the axial impulse turbine implemented in the acoustic impedance meter

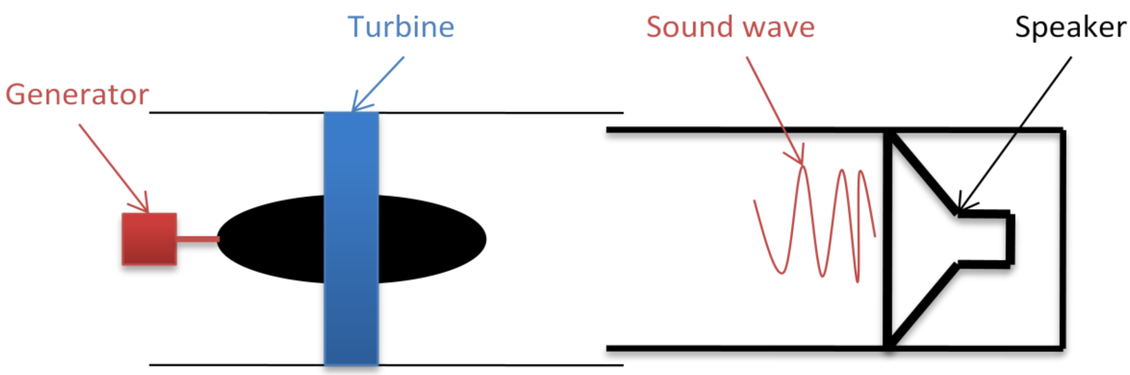


Figure 7.2: Schematical overview of axial impulse turbine and the acoustic impedance meter

Before the tests are performed a hypothesis is made to compare the theoretical calculations with the theory. For the hypothesis the same calculation method is used as in chapter 6, but with different input values. These values are given by Aster Thermoakoestische Systemen and shown in Table 7.1. The results of this hypothesis are shown in Table 7.2

Table 7.1: Input values for theroretical calculations under conditions in the acoustic impedance meter

Input	Symbol	Value	Unit
Average velocity	v_0	1-3	m/s
Temperature	T	296.5	K
Average pressure	p	98006	Pa
Specific gas constant	R	287	$J/kg \cdot K$
Density	ρ	1.152	kg/m^3

In this table can be seen that the tests are preformed under different conditions. The pressure is atmospheric (1 bar) instead of 4 bar. Besides, the velocity inputs are lower. Note, the input velocity shown in Table 7.1 is already the average velocity $\left(v_a \cdot \frac{1}{\sqrt{2}}\right)$ as mentioned earlier in 2.2. During the test the following parameters are measured:

- Velocity input
- Rotational speed
- Generator work
- Turbine work
- Wave frequency

Table 7.2: Results hypothesis

Input	Symbol	Value	Unit
Turbine work	W_D	0.4 – 3.7	W
Generator work	W_G	0.16 – 1.48	W
Rotational speed	n	351 – 1054	rpm

7.2. Turbine efficiency

The blue line in the Figure 7.3 shows the efficiency calculated in the hypothesis, which remains constant. This is because during the theoretical calculations, shown in chapter 6, the turbine efficiency depends on the guide vane inlet angle which is a fixed value. According to the test results the turbine efficiency is lower than is assumed theoretically, because not all efficiency losses occurring in practice are taken into account in the theoretical analysis. As can be seen in the red line in Figure 7.3, the turbine efficiency increases at higher velocity inputs. The reason why the efficiency drops after an average velocity of 2.5 m/s is probably an measurement error. This error is proven in the next sub-paragraph, including the reason why the efficiency increases at a higher velocity input. Note that the wave frequency of the test results shown in Figure 7.3 is 58.3 Hz.

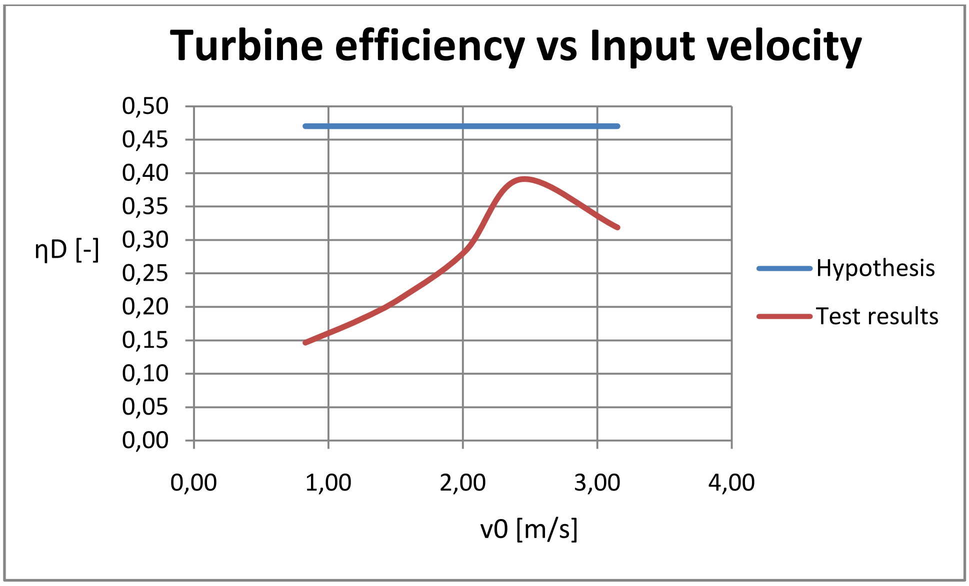


Figure 7.3: Turbine efficiency versus average velocity

7.3. Wave frequency

As discussed earlier in 6.6, according to theoretical assumptions the wave frequency should have an effect on the turbine efficiency. Below, in Figure 7.4 the same graph is made as in Figure 6.11 with the travelled distance versus the wave frequency. The difference is the average velocity v_0. The three different lines in the graph represent a velocity of 1, 2 and $3\,m/s$. As mentioned earlier in Table 6.1 the length of the turbine section is $135.58\,mm$. For, these low input velocities used during the test low wave frequencies are needed of less than $10\,Hz$.

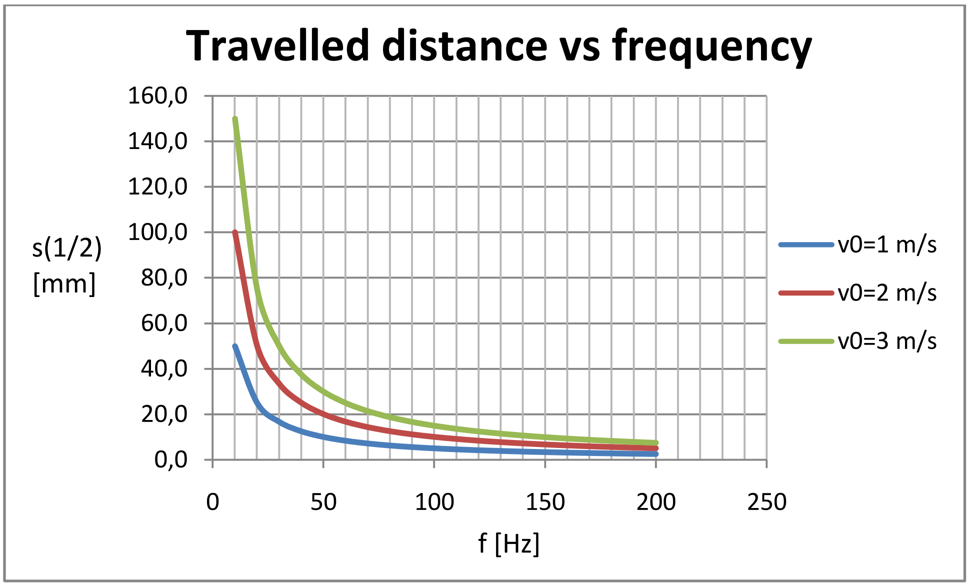


Figure 7.4: Travelled distance versus wave frequency at v0=1-3m/s

In Figure 7.5 two graphs are shown with the rotational speed versus the wave frequency on the left and on the right the turbine efficiency versus the rotational speed. One can assume that when the input velocity increases the rotational speed increases too. In the left graph of Figure 7.5 is proven that the wave frequency definitely has an effect on the rotational speed and finally on the turbine efficiency. So, during half a period of a wave a particle of air should pass through the entire turbine for an optimal efficiency.

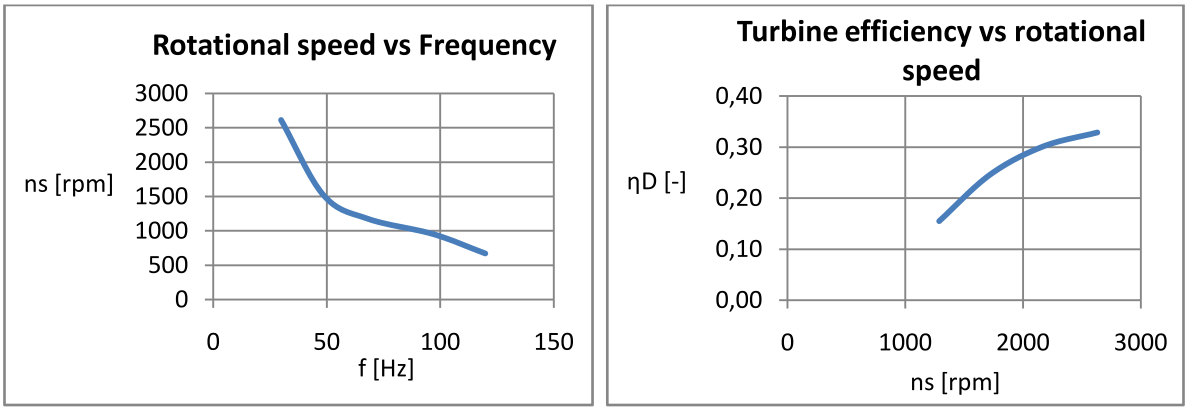


Figure 7.5: Rotational speed versus wave frequency (left), turbine efficiency versus rotational speed (right)

7.4. Rotational speed

In the left graph of Figure 7.5 can be seen that the rotational speed reached values over $2500\ rpm$. These values are way bigger than the rotational speed calculated theoretically. Therefore, a graph is shown in Figure 7.6 where the rotational speed per average velocity of the hypothesis (blue line) and the test results (red line) are shown.

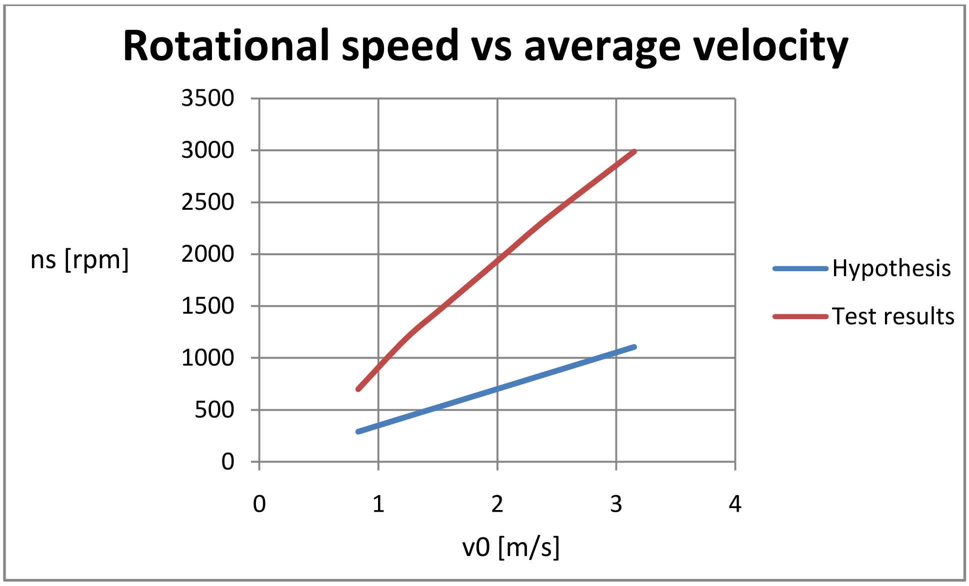


Figure 7.6: Rotational speed versus aveage velocity (Hypothesis and test results)

What is the reason for this big difference? Figure 7.7 shows a graph with the difference between the rotational speed values of the hypothesis and the test results. Concluding, there is a constant difference (factor) between the values of the hypothesis and test results. The average factor of the values shown in Figure 7.7 is ± 2.69.

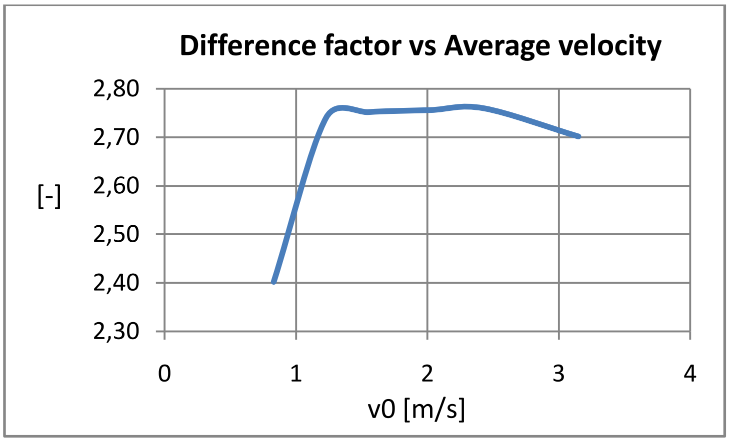


Figure 7.7: Difference factor between rotational speed of hypothesis and test results

According to the calculations shown in Appendix B the turbine could only reach rotational speeds up to $3000\,rpm$ when it is operating in the thermo-acoustic system with velocity amplitudes of $10 - 15\,m/s$. So, in the axial impulse turbine the speed increases even more as calculated for the final design, because in practice the airspeed increase in the guide vanes too.

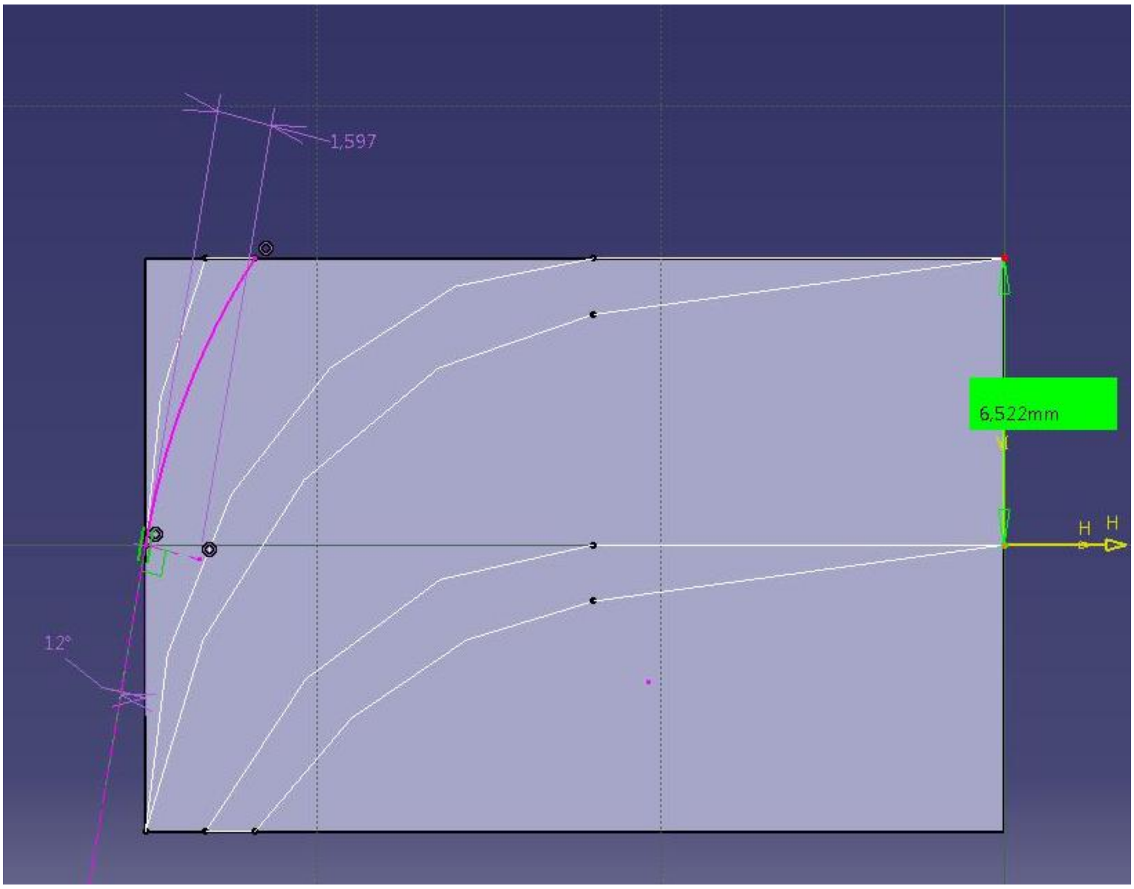


Figure 7.8: Cross-sectional area reduction in guide vane section

The cross-sectional are of the guide vanes seems to be the same at the front and aft. This can be seen in Figure 7.8 where the distance between two guide vanes is $6.522mm$ at the start.

Nevertheless, the moving direction of the air changes in vector direction until the $12°$ of the guide vane angle. At the moment where the air leaves the guide vane section, which is shown in Figure 7.8 at the left, the distance between two guide vanes is shorter $(1.597mm)$. This means that the speed is increasing even more as calculated before, namely with a factor: $4.08\left(=\frac{6.522}{1.597}\right)$. This factor can be compared with the factor shown in Figure 7.7, concluding that in practice there is a loss of efficiency (the difference between 4.08 and 2.69). Therefore, the calculations performed in Appendix B are adjusted. A calculation factor of 2.69 is added to the following formula:

$$v_1 = 2.69 \cdot \left(\frac{A_{tube}}{A_{turbine}} \cdot v_0\right)$$

Calculations are performed again and the results are shown in Figure 7.9. These results show that a better hypothesis of the rotational speed can be made when increasing airspeed at the guide vane section is taken into account.

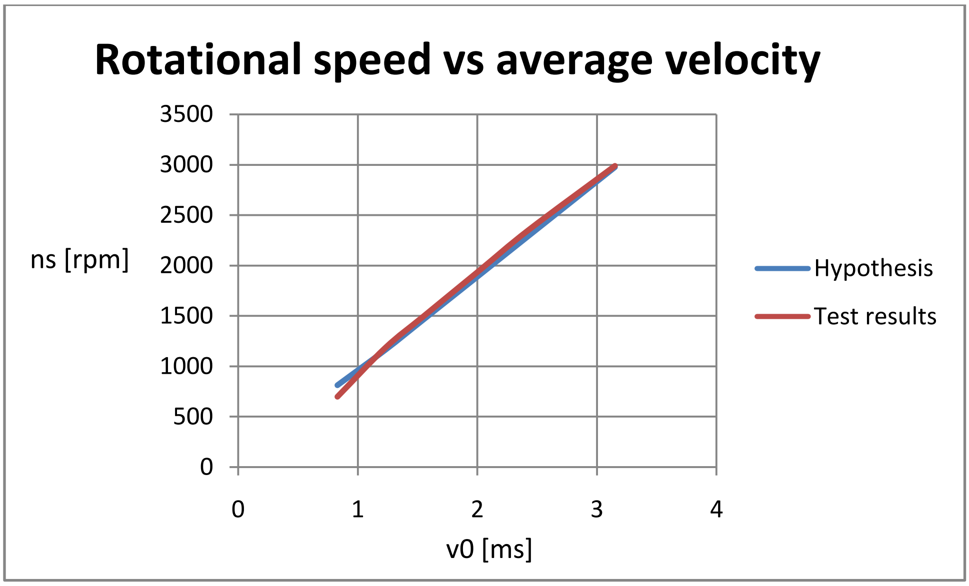


Figure 7.9: Rotational speed versus average velocity (hypothesis and test results)

7.5. Turbine work

Figure 7.10 results of the hypothesis and the test are shown in a graph. The results of the hypothesis are a slightly higher, but still it gives a good view of what is happening in practice. At a velocity input of $2.5\,m/s$ the rest results are higher than the hypothesis. This can be a result of a measurement error.

Overall, these test results agree with the theoretical calculations. Now it can be confirmed that an output power of 50 Watt of electricity is not possible. On the other hand, as shown in Figure 7.4 the wave frequency for an optimal turbine efficiency is $10\ Hz$, while a wave frequency of $58.3\ Hz$ is used during the test. This means that the efficiency, the turbine and generator work would be higher at an optimal frequency. To prove this new tests need to be performed.

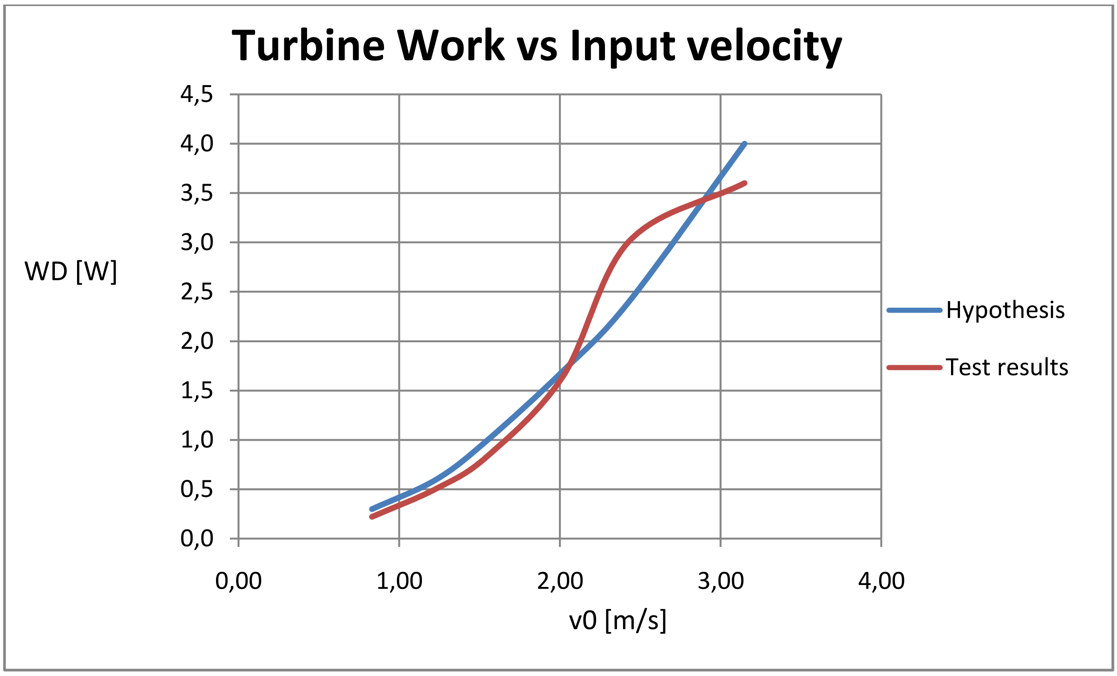


Figure 7.10: Turbine work versus average velocity

7.6. Conclusion

According to the hypothesis and the test results an output power of 50 Watt of electricity is not possible with this axial impulse turbine. Nevertheless, the wave frequency can be changed to increase the output power of the turbine and generator. Up to now an output power of $32.5\ W$ is possible. Test results show that the velocity in the turbine is increased by the guide vanes which is not taken into account during the theoretical calculations. During the tests the highest turbine efficiency reached is 40%. The turbine efficiency increases at higher velocity amplitudes, so a turbine efficiency of 47% as calculated theoretically can still be reached.

8. Conclusion

The best bi-directional turbine type to convert acoustic power into electricity is the axial impulse turbine, because this turbine is the most efficient. Moreover, the axial impulse turbine has the best starting characteristics and due to its configuration it is easy to implement into the thermo-acoustic system. A bi-directional turbine uses the moving air, caused by the sound waves, to convert the acoustic power into electricity.

The bi-directional turbine (axial impulse turbine) is a good alternative compared to the linear alternator. An axial impulse turbine is smaller than a linear alternator to generate the same amount of electricity. Moreover, the axial impulse turbine is cheaper than the linear alternator because the expansive material neodymium is not needed.

Theoretically, a turbine efficiency of 47% is possible. Nevertheless, tests have shown that in practice this efficiency is not possible. During these tests the maximum reached turbine efficiency reached is 40%, which is not under optimal conditions. According to the test results the turbine efficiency increases at higher input velocities. So, the turbine efficiency of the axial impulse turbine could increase when it is operating at optimal conditions where the input velocity is higher.

According to theoretical calculations an output power of 50 Watt of electricity is not possible, only $32.5\ W$. After analysis of test results it is found out that these results agree with the hypothesis. Nevertheless, reductions of energy losses in the axial impulse turbine can improve the output power of electricity. Besides, the tests are not performed at an optimal wave frequency.

Theoretical analysis and test results show that the wave frequency of the sound waves, produced by the thermo-acoustic system, definitely have an effect on the turbine efficiency. For every input velocity there is an optimal frequency which is about $70\ Hz$ when the axial impulse turbine is operating in the thermo-acoustic system under optimal conditions.

9. Recommendations

Up to now a output power of 50 Watt of electricity is not possible, however the output power can be increased with the following adjustments:

- First of all, the axial impulse turbine needs to be tested in the thermo-acoustic system. For the tests performed during this project the axial impulse turbine is implemented in another system which is less powerful. This is done because the thermo-acoustic system designed by Aster is not finished, which resulted in a lower output power of electricity. Therefore, tests need to be performed when the axial impulse turbine is implemented into the thermo-acoustic system.
- The tests are not performed with the optimal wave frequency. Due to this, the turbine efficiency is lower which results in a lower output power. Therefore, new tests need to be performed to find the right wave frequency for an optimal turbine efficiency. During these new test a higher output power of electricity can be reached.
- In the guide vanes energy is lost. According to the theoretical calculations the velocity should increase with a factor of around 4, while in practice this factor is about 2.7. Therefore, an analysis of the final design of the axial impulse turbine with Computational Fluid Dynamics (CFD) is recommended. With this analysis the places at the guide vanes can be found where energy is lost. Thereafter, the design of the axial impulse turbine can be optimized in order to reduce these losses.
- The used calculation method is not accurate enough, because most of the results does not agree with the theoretical calculations. Therefore, another calculation method needs to be used to give better predictions of what happens in practice. In several papers such as [7], [10], [13] and [14] another calculation method is used. Therefore, it is recommended to use this calculation method in order to find out if the results agree with the test results.

Besides these recommendations thermo-acoustic systems could be used for other applications. The thermo-acoustic system for this project is designed for small communities. The benefit of a thermo-acoustic system is that is only needs heat. Therefore, these systems can be applied to concentrators of solar energy systems (shown in Figure 9.1 at the left) or in flaring systems of factories (shown in Figure 9.1 at the right). In this way sustainable energy can be produced in an easy way.

Figure 9.1: Solar energy system with concentrator on the top (left), flaring system (right)

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Appendix

Appendix A

Afstudeeropdracht thermo-akoestische generator

Onderwerp

Een recente ontwikkeling is het gebruik van thermo-akoestiek voor biomassa-energie systemen. Thermo-akoestiek is een concept waarbij warmte wordt gebruikt om een geluidsgolf te genereren. Deze geluidsgolf kan worden omgezet in elektriciteit met behulp van een generator.

Opdracht omschrijving

In deze opdracht wordt gekeken naar bi-directionele turbines aangesloten op generatoren. Er zijn verschillende manieren om de generator aan te sturen, voorbeelden hiervan is een Wells-turbine een radiale en axiale impuls turbine. Door middel van een onderzoek naar deze technieken en een trade-off zal het beste concept worden uitgekozen en in detail in CATIA ontworpen worden. Het ontwerp wordt uiteindelijk met 3D print techniek gebouwd. Daarna wordt de turbine in een bestaande test opstelling getest en de resultaten vergeleken met het theoretisch model. Het doel is om uiteindelijk met de ontworpen generator ca. 50 Watt te genereren.

Stage periode

De afstudeerstage valt binnen de periode februari – juni 2012 en wordt uitgevoerd bij FACT-Foundation aan de Generaal Foulkersweg 9A te Wageningen. De afstudeerbegeleider vanuit FACT is dhr. Winfried Rijsenbeek

Competenties (Luchtvaarttechnologie)

- Maken van een detailontwerp
- Realiseren van een product
- Testen van een product
- Optimaliseren van een product

Verplichte competenties

- Opstellen van een projectplan
- Zelfsturing

Planning

- Literatuurstudie(1 maand)
- Ontwerp generator (2 maanden)
- Bestellen en opbouw prototype (0.5 maand)
- Testen prototype en verwerken en analyseren van testresultaten (1 maand)
- Optimaliseren (0.5 maand)

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Appendix B

First the cross-section areas in the tube need to be calculated, where A_0 is the cross-sectional area in the tube and A_1 at the turbine section:

$$A_0 = \pi \cdot r_0^2 = \pi \cdot 42.3^2$$

$$A_0 = 5621.2mm^2 = 5.6212 \cdot 10^{-3}\, m^2$$

$$A_1 = (\pi \cdot r_1^2) - (\pi \cdot r_2^2) = (\pi \cdot 36^2) - (\pi \cdot 25.2^2)$$

$$A_1 = 2076.5\, mm^2 = 2.0765 \cdot 10^{-3}\, m^2$$

The velocity amplitude $v_a = 14.09\, m/s$, because this is a peak value this velocity cannot be assumed as the constant speed flowing through the turbine. To get an average velocity the velocity amplitude an effective value is used [15]. Velocity v_0 is the speed at A_0.

$$v_0 = \frac{v_a}{\sqrt{2}}$$

$$v_0 = \frac{14.09}{\sqrt{2}}$$

$$v_0 = 9.96\ m/s$$

As a result of the duct velocity v_0 is increased to v_1. This is calculated by multiplying v_0 with the difference in cross-sectional areas between A_0 and A_1, the so called Bernoulli's equation [1].

$$v_1 = \frac{A_0}{A_1} \cdot v_0$$

$$v_1 = \frac{5.6212 \cdot 10^{-3}}{2.0765 \cdot 10^{-3}} \cdot 9.86$$

$$v_1 = 26.96\ m/s$$

The efficiency of the turbine is depended on the guide vane inlet angle α. For a most efficient turbine an angle of 12° is used:

$$\eta_D = 0.5 \cdot \left[\frac{\cos^2 \alpha}{2} \cdot (1 + k_b)\right]$$

$$\eta_D = 0.5 \cdot \left[\frac{\cos^2 (12°)}{2} \cdot (1 + 0.97)\right]$$

$$\eta_D = 0.47$$

The blade velocity is calculated which can never be more than the half of the value of the inlet velocity v_1.

$$v_b = v_1 \cdot \frac{\cos\alpha}{2}$$

$$v_b = 26.96 \cdot \frac{\cos\ (12°)}{2}$$

$$v_b = 13.19\, m/s$$

The tangential velocity Δv_ω:

$$\Delta v_\omega = \frac{\eta_D \cdot v_1^2}{2 \cdot v_b}$$

$$\Delta v_\omega = \frac{0.47 \cdot 26.96^2}{2 \cdot 13.19}$$

$$\Delta v_\omega = 12.99\, m/s$$

Up to now the absolute velocity v_1, the blade velocity v_b and the guide vane inlet angle α are known. These values are used to calculate the blade inlet angle β.

$$\beta = \tan^{-1}\left(\frac{v_1 \cdot \sin\alpha}{v_1 \cdot \cos\alpha - v_b}\right)$$

$$\beta = \tan^{-1}\left(\frac{26.96 \cdot \sin(12°)}{26.96 \cdot \cos\ (12°) - 13.19}\right)$$

$$\beta = 23.03°$$

The volume in the blade-section:

$$V = A_1 \cdot l_b$$

$$V = 2.0765 \cdot 10^{-3} \cdot 0.025$$

$$V = 5.2 \cdot 10^{-5}\, m^3$$

Mass of the air in the blade-section:

$$m = \rho \cdot V$$

$$m = 4.241 \cdot 5.2 \cdot 10^{-5}$$

$$m = 2.21 \cdot 10^{-4}\, kg$$

Time that the air needs to pass through the blade-section:

$$t = \frac{l_b}{v_1}$$

$$t = \frac{0.025}{26.7}$$

$$t = 9.3 \cdot 10^{-4}\ s$$

The mass flow rate can now be calculated, which can be used to calculate the tangential thrust of the turbine.

$$\dot{m} = \frac{m}{t}$$

$$\dot{m} = \frac{2.21 \cdot 10^{-4}}{9.3 \cdot 10^{-4}}$$

$$\dot{m} = 0.24\, kg/s$$

Tangential thrust:

$$P_t = \dot{m} \cdot \Delta v_\omega$$

$$P_t = 0.24 \cdot 12.99$$

$$P_t = 3.1\ N$$

The blading work, this is the work that is done by the turbine which is an important value. Finally, this value is used as an input for the output work of the generator [4].

$$W_D = P_t \cdot v_b$$

$$W_D = 3.1 \cdot 13.19$$

$$W_D = 40.7\ W$$

Besides, the blading work the energy that goes through the blades (E_b) is calculated too. This energy can be seen as a kinetic energy. The energy that goes through the blades is always higher than the work of the turbine, because there is always a loss in energy. This energy is calculated in the following way [4]:

$$E_b = \frac{1}{2} \cdot \dot{m} \cdot v_1^2$$

$$E_b = \frac{1}{2} \cdot 0.24 \cdot 26.96^2$$

$$E_b = 87.2\ W$$

An evaluation of the turbine efficiency is done to be sure that the calculations are done correctly. This is done by dividing the turbine work through the energy that goes through the blades [4].

$$\eta_D = \frac{W_D}{E_b}$$

$$\eta_D = \frac{40.7}{87.2}$$

$$\eta_D = 0.47$$

The circumference of the turbine:

$$\varphi = 2 \cdot \pi \cdot r_0$$

$$\varphi = 2 \cdot \pi \cdot 0.036$$

$$\varphi = 0.23\ m$$

Rotational speed of the turbine in rounds per minute:

$$n = \left(\frac{v_b}{\varphi}\right) \cdot 60$$

$$n = \left(\frac{13.19}{0.23}\right) \cdot 60$$

$$n = 3441\ rpm$$

After analyzing several generators it is found out that the average efficiency of a generator is around 80%. Now the output work of the generator can be calculated, which is 80% of the turbine work.

$$W_G = 0.8 \cdot W_D$$

$$W_G = 0.8 \cdot 40.7$$

$$W_G = 32.6\ W$$